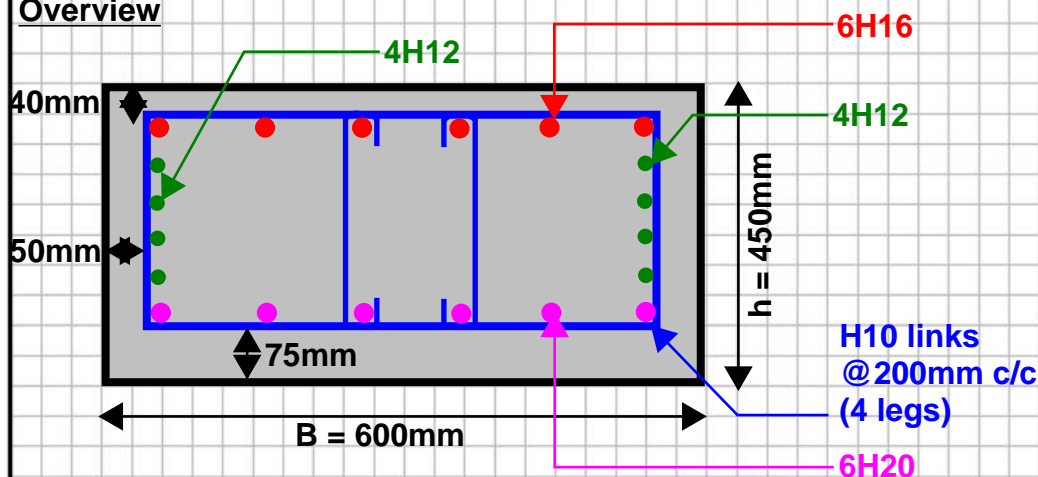


Overview**Beam Geometry**

Beam width $B = 600\text{mm}$
 Beam height $h = 450\text{mm}$
 Beam length $L = 3500\text{mm}$
 (simply supported)

Concrete Cover

Top Cover $c_{\text{top}} = 40\text{mm}$
 Bottom Cover $c_{\text{bottom}} = 75\text{mm}$
 Side Left Cover $c_{\text{sideL}} = 50\text{mm}$
 Side Right Cover $c_{\text{sideR}} = 50\text{mm}$

Concrete Properties - User Inputs

Concrete Grade: CUSTOM

Characteristic Compressive Cylinder Strength $f_{\text{ck}} = 27\text{N/mm}^2$

Aggregate Type: Limestone

Maximum Aggregate Size $a_g = 20\text{mm}$

Reinforcing Steel - User Inputs

Yield Strength of Reinforcing Steel $f_{yk} = 500\text{N/mm}^2$ (See BS EN1992-1-1 Section 3.2.2(3)P)

Modulus of Elasticity of Reinforcing Steel $E_s = 205,000\text{N/mm}^2$

Crack Widths - User Inputs

Allowable Crack Width $w_k = 0.3\text{mm}$ (See BS EN1992-1-1 Table 7.1N or BS EN1992-1-3)

Type of Rebar = High Bond Bars (for calculation of k_1 factor BS EN1992-1-1 Section 7.3.4)

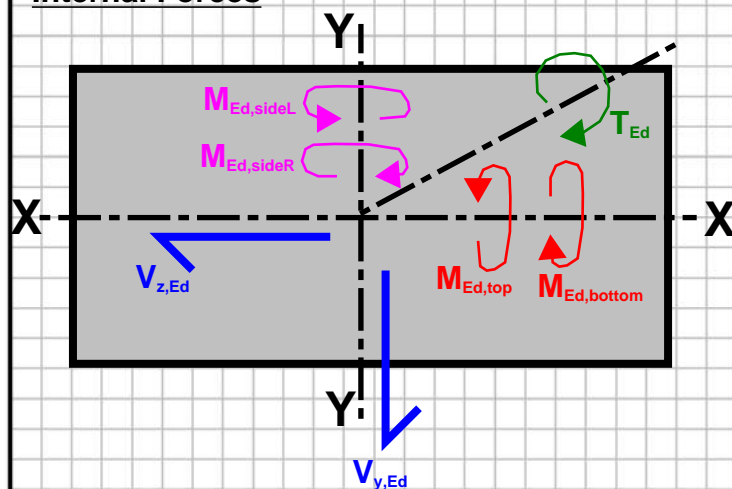
Type of Loading = Bending (See BS EN1992-1-1 7.3.4)

Number of days when considering short term loading $t = 20$ days

s factor for concrete type = 0.2

Creep factor for concrete at 28days $\phi_c = 1.857$

(2 is typical for creep but a more detailed assessment should be made where creep is a concern. To do this refer to BS EN1992-1-1 Appendix B and section 3.1.4)

Internal Forces

**Bottom Moments $M_{Ed,bottom} = 50\text{kNm ULS}$
 $= 35\text{kNm SLS}$**

**Top Moments $M_{Ed,top} = 25\text{kNm ULS}$
 $= 15\text{kNm SLS}$**

**Side Left Moments $M_{Ed,sideL} = 10\text{kNm ULS}$
 $= 5\text{kNm SLS}$**

**Side Right Moments $M_{Ed,sideR} = 10\text{kNm ULS}$
 $= 5\text{kNm SLS}$**

Major Axis Shear $V_{y,Ed} = 110\text{kN ULS}$

Minor Axis Shear $V_{z,Ed} = 55\text{kN ULS}$

Torsion $T_{Ed} = 7\text{kNm}$

Geometrical ChecksCheck Spacing Between Bottom Longitudinal Bars

Clear Distance between bottom longitudinal bars

$$= [\text{beam width} - \text{side cover left} - \text{side cover right} - \text{link diameter} - \text{link diameter} - (\text{no of bars} * \text{bar diameter})] / (\text{no of bars} - 1)$$

$$= [600\text{mm} - 50\text{mm} - 50\text{mm} - 10\text{mm} - 10\text{mm} - (6 \text{ bars} * 20\text{mm})] / (6 \text{ bars} - 1)$$

$$= 72\text{mm}$$

limiting clear distance between bottom longitudinal bars (BS EN1992-1-1 Section 8.2)

$$= \max\{ \text{bar diameter} , \text{max aggregate size} + 5\text{mm} , 20\text{mm} \}$$

$$= \max\{ 20\text{mm} , 20\text{mm} + 5\text{mm} , 20\text{mm} \}$$

$$= 25\text{mm}$$

The clear distance (72mm) is greater than the minimum clear distance (25mm) therefore the bar spacing is OK.

Check Spacing Between Top Longitudinal Bars

Clear Distance between top longitudinal bars

$$= [\text{beam width} - \text{side cover left} - \text{side cover right} - \text{link diameter} - \text{link diameter} - (\text{no of bars} * \text{bar diameter})] / (\text{no of bars} - 1)$$

$$= [600\text{mm} - 50\text{mm} - 50\text{mm} - 10\text{mm} - 10\text{mm} - (6 \text{ bars} * 16\text{mm})] / (6 \text{ bars} - 1)$$

$$= 76.8\text{mm} \text{ (rounded to } 77\text{mm)}$$

limiting clear distance between top longitudinal bars (BS EN1992-1-1 Section 8.2)

$$= \max\{ \text{bar diameter} , \text{max aggregate size} + 5\text{mm} , 20\text{mm} \}$$

$$= \max\{ 16\text{mm} , 20\text{mm} + 5\text{mm} , 20\text{mm} \}$$

$$= 25\text{mm}$$

The clear distance (77mm) is greater than the minimum clear distance (25mm) therefore the bar spacing is OK.

Check Spacing Between Side Left Longitudinal Bars

The clear distance between the side bars is more tricky to determine because it is affected by the top and bottom longitudinal steelwork. We will assume that the side bars are spaced equidistantly based on their centreline positions.

In this instance the minimum clear distance between side bars occurs between the bottom H20 longitudinal bar (pink) and the first H12 side bar (green)

$$\text{Centreline bar spacing} = [450\text{mm} - 40\text{mm cover} - 75\text{mm cover} - 10\text{mm link} - 10\text{mm link} - (16\text{mm top bar} / 2) - (20\text{mm bottom bar} / 2)] / (4 \text{ side bars} + 1)$$

$$\text{Centreline bar spacing} = 59.4\text{mm}$$

$$\text{Minimum clear distance} = 59.4\text{mm} - (20\text{mm bottom bar} / 2) - (12\text{mm side bar} / 2)$$

$$= 43.4\text{mm}$$

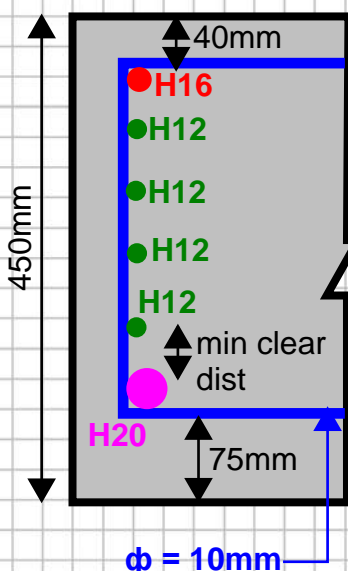
limiting clear distance between side bars (BS EN1992-1-1 Section 8.2)

$$= \max\{ \text{bar diameter} , \text{max aggregate size} + 5\text{mm} , 20\text{mm} \}$$

$$= \max\{ 12\text{mm} , 20\text{mm} + 5\text{mm} , 20\text{mm} \}$$

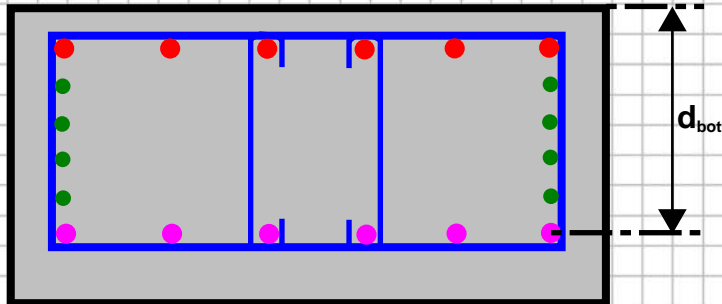
$$= 25\text{mm}$$

The clear distance (43.3mm) is greater than the minimum clear distance (25mm) therefore the bar spacing is OK.



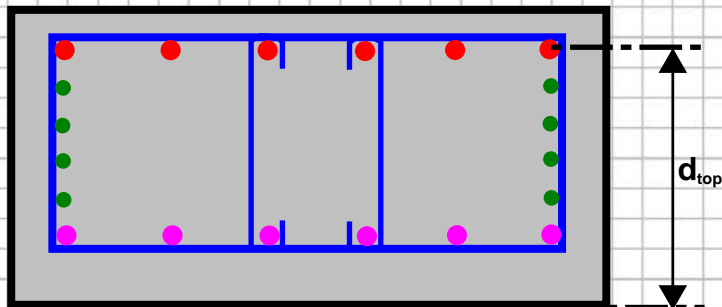
Check Spacing Between Side Right Longitudinal Bars

The side right longitudinal bars use the same reinforcement as the left longitudinal bars so the calculation is the same as before

Find Depths to ReinforcementDistance to Bottom Reinforcement

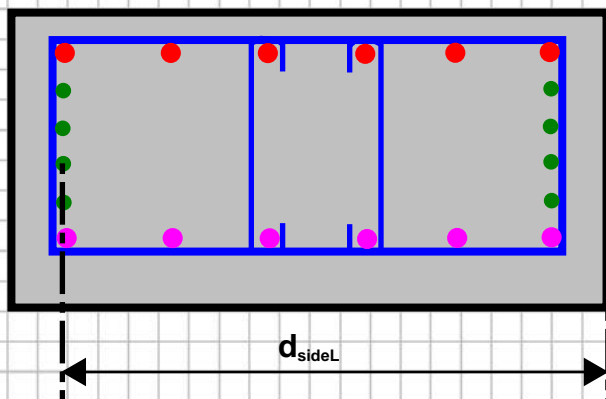
$$d_{\text{bot}} = 450\text{mm beam depth} - 75\text{mm bottom cover} - 10\text{mm link diameter} - (20\text{mm bottom steel diameter} / 2)$$

$$d_{\text{bot}} = 355\text{mm}$$

Distance to Top Reinforcement

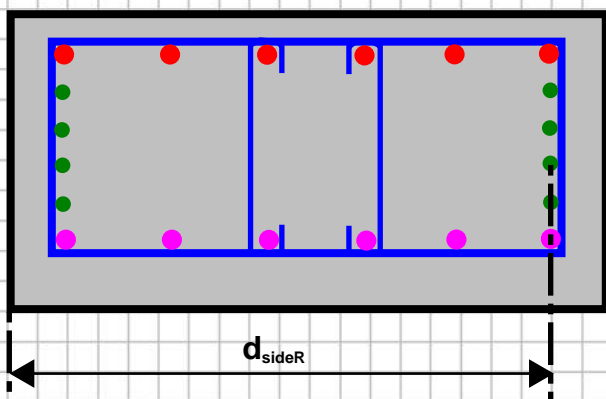
$$d_{\text{top}} = 450\text{mm beam depth} - 40\text{mm top cover} - 10\text{mm link diameter} - (16\text{mm top steel diameter} / 2)$$

$$d_{\text{top}} = 392\text{mm}$$

Distance to Side Left Reinforcement

$$d_{\text{sideL}} = 600\text{mm beam width} - 50\text{mm side left cover} - 10\text{mm link diameter} - (12\text{mm side left steel diameter} / 2)$$

$$d_{\text{sideL}} = 534\text{mm}$$

Distance to Side Right Reinforcement

$$d_{\text{sideR}} = 600\text{mm beam width} - 50\text{mm side right cover} - 10\text{mm link diameter} - (12\text{mm side right steel diameter} / 2)$$

$$d_{\text{sideR}} = 534\text{mm}$$

Check Maximum Moment Capacity of BeamBottom Moments

$$M_{\max} = 0.167 * f_{ck} * B * d_{\text{bot}}^2$$

$$M_{\max} = 0.167 * 27\text{N/mm}^2 * 600\text{mm} * (355\text{mm})^2$$

$$M_{\max} = 340948035\text{Nmm} = 340.9\text{kNm}$$

340.9kNm > 50kNm applied moment @ ULS --> OK we can continue designing as singly reinforced (i.e. no compression reinforcement required).

Top Moments

$$M_{\max} = 0.167 * f_{ck} * B * d_{\text{top}}^2$$

$$M_{\max} = 0.167 * 27\text{N/mm}^2 * 600\text{mm} * (392\text{mm})^2$$

$$M_{\max} = 415722585.6\text{Nmm} = 415.7\text{kNm}$$

415.7kNm > 25kNm applied moment @ ULS --> OK we can continue designing as singly reinforced (i.e. no compression reinforcement required).

Side Left Moments

$$M_{\max} = 0.167 * f_{ck} * h * d_{\text{sideL}}^2$$

$$M_{\max} = 0.167 * 27\text{N/mm}^2 * 450\text{mm} * (534\text{mm})^2$$

$$M_{\max} = 578595781.8\text{Nmm} = 578.6\text{kNm}$$

578.6kNm > 10kNm applied moment @ ULS --> OK we can continue designing as singly reinforced (i.e. no compression reinforcement required).

Side Right Moments

$$M_{\max} = 0.167 * f_{ck} * h * d_{\text{sideR}}^2$$

$$M_{\max} = 0.167 * 27\text{N/mm}^2 * 450\text{mm} * (534\text{mm})^2$$

$$M_{\max} = 578595781.8\text{Nmm} = 578.6\text{kNm}$$

578.6kNm > 10kNm applied moment @ ULS --> OK we can continue designing as singly reinforced (i.e. no compression reinforcement required).

Calculated Concrete Properties

Mean concrete compressive cylinder strength $f_{cm} = f_{ck} + 8\text{N/mm}^2 = 27\text{N/mm}^2 + 8\text{N/mm}^2 = 35\text{N/mm}^2$
(BS EN1992-1-1 Table 3.1)

Mean concrete tensile strength $f_{ctm} = 0.3f_{ck}^{2/3}$ for concrete grades \leq C50/60
 $f_{ctm} = 2.12\ln(1 + f_{cm}/10)$ for concrete grades $>$ C50/60

$$f_{ctm} = 0.3 * (27\text{N/mm}^2)^{2/3} = 2.7\text{N/mm}^2$$

(BS EN1992-1-1 Table 3.1)

Mean Modulus of Elasticity $E_{cm, \text{basic}} = 22 * (f_{cm} * 0.1)^{0.3}$

$$E_{cm, \text{basic}} = 22 * (35\text{N/mm}^2 * 0.1)^{0.3}$$

$$E_{cm, \text{basic}} = 32.03\text{GPa} = 32036\text{N/mm}^2$$

(BS EN1992-1-1 Table 3.1)

Type of aggregate used is limestone therefore multiply by 0.9 (10% reduction)

$$E_{cm} = E_{cm, \text{basic}} * 0.9 = 32036\text{N/mm}^2 * 0.9 = 28832\text{N/mm}^2$$

(BS EN1992-1-1 Section 3.1.3(2))

Check Moment Capacity of BeamBottom Moments

$$K \text{ factor} = M_{\text{bottom}} / (f_{ck} * B * d_{\text{bot}}^2)$$

$$K \text{ factor} = 50\text{kNm} / (27\text{N/mm}^2 * 600\text{mm} * (355\text{mm})^2)$$

$$K \text{ factor} = 0.0245$$

$$\text{Lever arm } Z = \min\{ 0.95d_{\text{bot}}, d_{\text{bot}} * [0.5 + \sqrt{0.25 - 3K/3.4}] \}$$

$$\text{Lever arm } Z = \min\{ 0.95 * 355\text{mm}, 355\text{mm} * [0.5 + \sqrt{0.25 - 3*0.0245/3.4}] \}$$

$$\text{Lever arm } Z = \min\{ 337.25\text{mm}, 347.15\text{mm} \}$$

$$\text{Lever arm } Z = 337.25\text{mm}$$

$$\text{Minimum area of reinforcement } A_{s,\text{min}} = \max\{ 0.26f_{ctm}/f_{yk}, 0.0013 \} * B * d_{\text{bot}}$$

$$\text{Minimum area of reinforcement } A_{s,\text{min}} = \max\{ 0.26 * 2.7\text{N/mm}^2 / 500\text{N/mm}^2, 0.0013 \} * 600\text{mm} * 355\text{mm}$$

$$\text{Minimum area of reinforcement } A_{s,\text{min}} = 299.1\text{mm}^2$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = \max\{ M_{\text{bottom}} / [0.87 * f_{yk} * z], A_{s,\text{min}} \}$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = \max\{ 50\text{kNm} / [0.87 * 500\text{N/mm}^2 * 337.25\text{mm}], 299.1\text{mm}^2 \}$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = 340.8\text{mm}^2$$

$$\text{Area of steel provided } A_{s,\text{prov}} = 6H20 = [(20\text{mm})^2 * \pi * (1/4)] * 6 \text{ bars} = 1885\text{mm}^2$$

$$\text{Utilisation} = A_{s,\text{req}} / A_{s,\text{prov}} = 340.8\text{mm}^2 / 1885\text{mm}^2 = 18\% \rightarrow \text{OK}$$

Top Moments

$$K \text{ factor} = M_{\text{top}} / (f_{ck} * B * d_{\text{top}}^2)$$

$$K \text{ factor} = 25\text{kNm} / (27\text{N/mm}^2 * 600\text{mm} * (392\text{mm})^2)$$

$$K \text{ factor} = 0.0100$$

$$\text{Lever arm } Z = \min\{ 0.95d_{\text{top}}, d_{\text{top}} * [0.5 + \sqrt{0.25 - 3K/3.4}] \}$$

$$\text{Lever arm } Z = \min\{ 0.95 * 392\text{mm}, 392\text{mm} * [0.5 + \sqrt{0.25 - 3*0.0100/3.4}] \}$$

$$\text{Lever arm } Z = \min\{ 372.4\text{mm}, 388.5\text{mm} \}$$

$$\text{Lever arm } Z = 372.4\text{mm}$$

$$\text{Minimum area of reinforcement } A_{s,\text{min}} = \max\{ 0.26f_{ctm}/f_{yk}, 0.0013 \} * B * d_{\text{top}}$$

$$\text{Minimum area of reinforcement } A_{s,\text{min}} = \max\{ 0.26 * 2.7\text{N/mm}^2 / 500\text{N/mm}^2, 0.0013 \} * 600\text{mm} * 392\text{mm}$$

$$\text{Minimum area of reinforcement } A_{s,\text{min}} = 330.2\text{mm}^2$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = \max\{ M_{\text{top}} / [0.87 * f_{yk} * z], A_{s,\text{min}} \}$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = \max\{ 25\text{kNm} / [0.87 * 500\text{N/mm}^2 * 372.4\text{mm}], 330.2\text{mm}^2 \}$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = 330.2\text{mm}^2$$

$$\text{Area of steel provided } A_{s,\text{prov}} = 6H16 = [(16\text{mm})^2 * \pi * (1/4)] * 6 \text{ bars} = 1206.4\text{mm}^2$$

$$\text{Utilisation} = A_{s,\text{req}} / A_{s,\text{prov}} = 330.2\text{mm}^2 / 1206.4\text{mm}^2 = 27\% \rightarrow \text{OK}$$

Side Left Moments

$$K \text{ factor} = M_{\text{sideL}} / (f_{\text{ck}} * h * d_{\text{sideL}}^2)$$

$$K \text{ factor} = 10\text{kNm} / (27\text{N/mm}^2 * 450\text{mm} * (534\text{mm})^2)$$

$$K \text{ factor} = 0.0029$$

$$\text{Lever arm } Z = \min\{0.95d_{\text{sideL}}, d_{\text{sideL}} * [0.5 + \sqrt{0.25 - 3K/3.4}]\}$$

$$\text{Lever arm } Z = \min\{0.95 * 534\text{mm}, 534\text{mm} * [0.5 + \sqrt{0.25 - 3 * 0.0029/3.4}]\}$$

$$\text{Lever arm } Z = \min\{507.3\text{mm}, 532.6\text{mm}\}$$

$$\text{Lever arm } Z = 507.3\text{mm}$$

$$\text{Minimum area of reinforcement } A_{s,\min} = \max\{0.26f_{\text{ctm}}/f_{\text{yk}}, 0.0013\} * h * d_{\text{sideL}}$$

$$\text{Minimum area of reinforcement } A_{s,\min} = \max\{0.26 * 2.7\text{N/mm}^2 / 500\text{N/mm}^2, 0.0013\} * 450\text{mm} * 534\text{mm}$$

$$\text{Minimum area of reinforcement } A_{s,\min} = 337.4\text{mm}^2$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = \max\{M_{\text{sideL}} / [0.87 * f_{\text{yk}} * z], A_{s,\min}\}$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = \max\{10\text{kNm} / [0.87 * 500\text{N/mm}^2 * 507.3\text{mm}], 337.4\text{mm}^2\}$$

$$\text{Area of reinforcement required } A_{s,\text{req}} = 337.4\text{mm}^2$$

$$\text{Area of steel provided } A_{s,\text{prov}} = 4\text{H}12 = [(12\text{mm})^2 * \pi * (1/4)] * 4 \text{ bars} = 452.4\text{mm}^2$$

$$\text{Utilisation} = A_{s,\text{req}} / A_{s,\text{prov}} = 337.4\text{mm}^2 / 452.4\text{mm}^2 = 75\% \rightarrow \text{OK}$$

Side Right Moments

Same as Side Left Moments - see above

Check Maximum Area of Reinforcement

$$\text{Maximum Area of Steel } A_{s,\max} = 0.04 * B * h$$

$$A_{s,\max} = 0.04 * 600\text{mm} * 450\text{mm}$$

$$A_{s,\max} = 10800\text{mm}^2$$

Check Bottom Steel

$$A_{s,\text{prov}} = 6\text{H}20 = 1885\text{mm}^2$$

$$\text{Utilisation} = A_{s,\text{prov}} / A_{s,\max} = 1885\text{mm}^2 / 10800\text{mm}^2 = 17\% \rightarrow \text{OK}$$

Check Top Steel

$$A_{s,\text{prov}} = 6\text{H}16 = 1206.4\text{mm}^2$$

$$\text{Utilisation} = A_{s,\text{prov}} / A_{s,\max} = 1206.4\text{mm}^2 / 10800\text{mm}^2 = 11\% \rightarrow \text{OK}$$

Check Side Left Steel

$$A_{s,\text{prov}} = 4\text{H}12 = 452.4\text{mm}^2$$

$$\text{Utilisation} = A_{s,\text{prov}} / A_{s,\max} = 452.4\text{mm}^2 / 10800\text{mm}^2 = 4\% \rightarrow \text{OK}$$

Check Side Right Steel

$$A_{s,\text{prov}} = 4\text{H}12 = 452.4\text{mm}^2$$

$$\text{Utilisation} = A_{s,\text{prov}} / A_{s,\max} = 452.4\text{mm}^2 / 10800\text{mm}^2 = 4\% \rightarrow \text{OK}$$

Check Deflection of Beam (L/d Ratio Check)

K Factor for Structural System

Support Arrangement = "Simply Supported"

K Factor = 1.0

BS EN1992-1-1 Table 7.4N

Reference Reinforcement Ratio

Reference reinforcement ratio $\rho_0 = 10^{-3} \sqrt{f_{ck}}$ Reference reinforcement ratio $\rho_0 = 10^{-3} \sqrt{27 \text{ N/mm}^2}$ Reference reinforcement ratio $\rho_0 = 0.00520$

BS EN1992-1-1 Section 7.4.2

Bottom Moments

Required Tension Reinforcement Ratio $\rho = A_{s, \text{req}} / (B * d_{\text{bot}})$ Required Tension Reinforcement Ratio $\rho = 340.8 \text{ mm}^2 / (600 \text{ mm} * 355 \text{ mm})$ Required Tension Reinforcement Ratio $\rho = 0.0016$ Required Compression Reinforcement Ratio $\rho' = 0$ (no compression reinforcement required)

$$\frac{L}{d} \text{ basic} = K \left[11 + 1.5 \sqrt{f_{ck}} \frac{\rho_0}{\rho} + 3.2 \sqrt{f_{ck}} \left(\frac{\rho_0}{\rho} - 1 \right)^{\frac{3}{2}} \right] \quad \text{for : } \rho \leq \rho_0$$

$$\frac{L}{d} \text{ basic} = K \left[11 + 1.5 \sqrt{f_{ck}} \frac{\rho_0}{\rho - \rho'} + \frac{1}{12} \sqrt{f_{ck}} \sqrt{\frac{\rho'}{\rho_0}} \right] \quad \text{for : } \rho > \rho_0 \quad \text{BS EN1992-1-1 Eq 7.16a \& 7.16b}$$

In this case $\rho < \rho_0 \rightarrow 0.0016 < 0.00520$

therefore use equation 7.16a

$$\frac{L}{d} \text{ basic} = 1.0 \left[11 + 1.5 \sqrt{27 \text{ N/mm}^2} \frac{0.0052}{0.0016} + 3.2 \sqrt{27 \text{ N/mm}^2} \left(\frac{0.0052}{0.0016} - 1 \right)^{\frac{3}{2}} \right]$$

L/d basic = 92.45

Stress in the Steel $\sigma_s = (310 * f_{yk} * A_{s, \text{req}}) / (500 * A_{s, \text{prov}})$ Stress in the Steel $\sigma_s = (310 * 500 \text{ N/mm}^2 * 340.8 \text{ mm}^2) / (500 * 1885 \text{ mm}^2)$ Stress in the Steel $\sigma_s = 56.046 \text{ N/mm}^2$

BS EN1992-1-1 Eq 7.17

Additional Factor for long spanning beams $\gamma_{\text{LONG}} = 1.0$ as beam span $\leq 7 \text{ m}$ (when beam span is greater than 7m $\gamma_{\text{LONG}} = 7 \text{ m} / \text{beam span in metres}$)

BS EN1992-1-1 Section 7.4.2

Allowable L/d = $\gamma_{\text{LONG}} * L/d \text{ basic} * (310 / \sigma_s)$ Allowable L/d = $1.0 * 92.45 * (310 / 56.046 \text{ N/mm}^2)$

Allowable L/d = 511.35

Actual L/d = beam span / depth to reinforcement

Actual L/d = 3500mm beam span / 355mm

Actual L/d = 9.85

Utilisation = actual L/d / Allowable L/d = $9.85 / 511.35 = 2\% \rightarrow \text{OK}$

Top Moments

Required Tension Reinforcement Ratio $\rho = A_{s,req} / (B * d_{top})$

Required Tension Reinforcement Ratio $\rho = 330.2\text{mm}^2 / (600\text{mm} * 392\text{mm})$

Required Tension Reinforcement Ratio $\rho = 0.00140$

Required Compression Reinforcement Ratio $\rho' = 0$ (no compression reinforcement required)

$$\frac{L}{d} basic = K[11 + 1.5\sqrt{f_{ck}} \frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}} (\frac{\rho_0}{\rho} - 1)^{\frac{3}{2}}] \quad \text{for : } \rho \leq \rho_0$$

$$\frac{L}{d} basic = K[11 + 1.5\sqrt{f_{ck}} \frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}} \sqrt{\frac{\rho'}{\rho_0}}] \quad \text{for : } \rho > \rho_0 \quad \text{BS EN1992-1-1 Eq 7.16a \& 7.16b}$$

In this case $\rho < \rho_0 \rightarrow 0.0014 < 0.00520$

therefore use equation 7.16a

$$\frac{L}{d} basic = 1.0[11 + 1.5\sqrt{27\text{N/mm}^2} \frac{0.0052}{0.0014} + 3.2\sqrt{27\text{N/mm}^2} (\frac{0.0052}{0.0014} - 1)^{\frac{3}{2}}]$$

$$L/d \text{ basic} = 114.3$$

$$\text{Stress in the Steel } \sigma_s = (310 * f_{yk} * A_{s,req}) / (500 * A_{s,prov})$$

$$\text{Stress in the Steel } \sigma_s = (310 * 500\text{N/mm}^2 * 330.2\text{mm}^2) / (500 * 1206.4\text{mm}^2)$$

$$\text{Stress in the Steel } \sigma_s = 84.85\text{N/mm}^2$$

BS EN1992-1-1 Eq 7.17

Additional Factor for long spanning beams $\gamma_{LONG} = 1.0$ as beam span $< 7\text{m}$

(when beam span is greater than 7m $\gamma_{LONG} = 7\text{m} / \text{beam span in metres}$)

BS EN1992-1-1 Section 7.4.2

$$\text{Allowable } L/d = \gamma_{LONG} * L/d \text{ basic} * (310 / \sigma_s)$$

$$\text{Allowable } L/d = 1.0 * 114.3 * (310 / 84.85\text{N/mm}^2)$$

$$\text{Allowable } L/d = 417.6$$

Actual L/d = beam span / depth to reinforcement

Actual L/d = 3500mm beam span / 392mm

Actual L/d = 8.93

Utilisation = actual L/d / Allowable L/d = $8.93 / 417.6 = 2\% \rightarrow \text{OK}$

Side Left Moments

Required Tension Reinforcement Ratio $\rho = A_{s,req} / (h * d_{sideL})$

Required Tension Reinforcement Ratio $\rho = 337.4\text{mm}^2 / (450\text{mm} * 534\text{mm})$

Required Tension Reinforcement Ratio $\rho = 0.00140$

Required Compression Reinforcement Ratio $\rho' = 0$ (no compression reinforcement required)

$$\frac{L}{d} basic = K[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}(\frac{\rho_0}{\rho} - 1)^{\frac{3}{2}}] \quad \text{for : } \rho \leq \rho_0$$

$$\frac{L}{d} basic = K[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}] \quad \text{for : } \rho > \rho_0 \quad \text{BS EN1992-1-1 Eq 7.16a \& 7.16b}$$

In this case $\rho < \rho_0 \rightarrow 0.0014 < 0.00520$

therefore use equation 7.16a

$$\frac{L}{d} basic = 1.0[11 + 1.5\sqrt{27\text{N/mm}^2}\frac{0.0052}{0.0014} + 3.2\sqrt{27\text{N/mm}^2}(\frac{0.0052}{0.0014} - 1)^{\frac{3}{2}}]$$

L/d basic = 114.3

Stress in the Steel $\sigma_s = (310 * f_{yk} * A_{s,req}) / (500 * A_{s,prov})$

Stress in the Steel $\sigma_s = (310 * 500\text{N/mm}^2 * 337.4\text{mm}^2) / (500 * 452.4\text{mm}^2)$

Stress in the Steel $\sigma_s = 231.2\text{N/mm}^2$

Additional Factor for long spanning beams $\gamma_{LONG} = 1.0$ as beam span $\leq 7\text{m}$

(when beam span is greater than 7m $\gamma_{LONG} = 7\text{m} / \text{beam span in metres}$)

BS EN1992-1-1 Section 7.4.2

Allowable L/d = $\gamma_{LONG} * L/d \text{ basic} * (310 / \sigma_s)$

Allowable L/d = $1.0 * 114.3 * (310 / 231.2\text{N/mm}^2)$

Allowable L/d = 153.3

Actual L/d = beam span / depth to reinforcement

Actual L/d = 3500mm beam span / 534mm

Actual L/d = 6.55

Utilisation = actual L/d / Allowable L/d = $6.55 / 153.3 = 4\% \rightarrow \text{OK}$

Side Right Moments

This is the same as the side left moments - see above

Check Shear

Partial Material Safety Factor for Concrete in Shear

Partial Factor $\gamma_c = 1.5$

BS EN1992-1-1 Table 2.1N

 $C_{R,dc}$ Factor $C_{R,dc} = 0.18 / \gamma_c = 0.18 / 1.5 = 0.12$

UK NA to BS EN1992-1-1 6.2.2(1)

Major Axis Shear - Bottom Steel $k \text{ factor} = 1 + \sqrt{[200 / d_{bot}]} \leq 2.0$ $k \text{ factor} = 1 + \sqrt{[200 / 355\text{mm}]} \leq 2.0$ $k \text{ factor} = 1.75 \leq 2.0 \rightarrow 1.75$

BS EN1992-1-1 6.2.2

Reinforcement ratio $\rho_l = A_{s,prov} / (B * d)$ Reinforcement ratio $\rho_l = 1885\text{mm}^2 / (600\text{mm} * 355\text{mm})$ Reinforcement ratio $\rho_l = 0.00885$

BS EN1992-1-1 6.2.2

 $v_{min} = 0.035k^{3/2} * f_{ck}^{-1/2}$ $v_{min} = 0.035(1.75)^{3/2} * (27\text{N/mm}^2)^{1/2}$ $v_{min} = 0.421$

UK NA to BS EN1992-1-1 6.2.2(1) and BS EN1992-1-1 Eq 6.3N

Shear Strength of Concrete $V_{Rd,c} = \max\{C_{Rd,c} * k * (100\rho_l * f_{ck})^{1/3}, v_{min}\} * B * d_{bot}$ Shear Strength of Concrete $V_{Rd,c} = \max\{0.12 * 1.75 * (100 * 0.00885 * 27\text{N/mm}^2)^{1/3}, 0.421\} * 600\text{mm} * 355\text{mm}$ Shear Strength of Concrete $V_{Rd,c} = 128.83\text{kN}$

BS EN1992-1-1 Equation 6.2a and 6.2b

Cross Sectional Area of Shear Reinforcement $A_{sw} = (10\text{mm})^2 * \pi * (1/4) * 4 \text{ legs} = 314.16\text{mm}^2$ Strength reduction factor for concrete cracked in shear $v_1 = v = 0.6[1 - (f_{ck} / 250)]$ Strength reduction factor for concrete cracked in shear $v_1 = v = 0.6[1 - (27\text{N/mm}^2 / 250)]$ Strength reduction factor for concrete cracked in shear $v_1 = v = 0.5352$

BS EN1992-1-1 Equation 6.6N and the UK National Annex

Coefficient for state of stress in the compression chord $\alpha_{cw} = 1.0$ (no pre-stress applied)

BS EN1992-1-1 Section 6.2.3(3) Note 3

Angle between the concrete strut and beam axis $\Theta = 21.8^\circ$ $\theta = \min(\max[\frac{1}{2}\sin^{-1}(\min[\frac{2V_{Ed}}{\alpha_{cw}f_{cwd}v_1}, 1]), 21.8^\circ], 45^\circ)$ Where $v_{Ed} = V_{Ed} / (B * Z)$ Lever arm Z (from previous moment calculation) $Z = 337.25\text{mm}$ (if no moment is added use $Z = 0.9d$)Design Strength of concrete in compression $f_{cwd} = \alpha_{ccw} * f_{ck} * (1 / \gamma_c)$ Design Strength of concrete in compression $f_{cwd} = 1.0 * 27\text{N/mm}^2 * (1 / 1.5)$ Design Strength of concrete in compression $f_{cwd} = 18\text{N/mm}^2$ $\alpha_{ccw} = 1.0$ from UK National Annex to BS EN1992-1-1

BS EN1992-1-1 Equation 3.15

Maximum Shear Capacity of Beam $V_{Rd,max} = [\alpha_{cw} * B * Z * v_1 * f_{cwd}] / [\cot(45^\circ) + \tan(45^\circ)]$ Maximum Shear Capacity of Beam $V_{Rd,max} = [1.0 * 600\text{mm} * 337.25\text{mm} * 0.5352 * 18\text{N/mm}^2] / 2$ Maximum Shear Capacity of Beam $V_{Rd,max} = 974.67\text{kN}$

BS EN1992-1-1 Equation 6.9

 $V_{y,Ed} < V_{Rd,max} \rightarrow 110\text{kN} < 974.67\text{kN} \rightarrow$ Shear design is possible

Partial Material Factor for shear links $\gamma_s = 1.15$

BS EN1992-1-1 Table 2.1N

Design yield strength of shear reinforcement $f_{ywd} = f_{yk} / \gamma_s$

Design yield strength of shear reinforcement $f_{ywd} = 500\text{N/mm}^2 / 1.15$

Design yield strength of shear reinforcement $f_{ywd} = 434.8\text{N/mm}^2$

Capacity of current shear link arrangement $V_{Rd,s} = (A_{sw} / \text{link spacing}) * Z * f_{ywd} * \cot(\Theta)$

Capacity of current shear link arrangement $V_{Rd,s} = (314.16\text{mm}^2 / 200\text{mm}) * 337.25\text{mm} * 434.8\text{N/mm}^2 * \cot(21.8^\circ)$

Capacity of current shear link arrangement $V_{Rd,s} = 575.9\text{kN}$

Utilisation = $V_{y,Ed} / V_{Rd,s} = 110\text{kN} / 575.9\text{kN} = 19\% \rightarrow \text{OK}$

Ratio of Shear Reinforcement $\rho_w = A_{sw} / (\text{link spacing} * B * \sin(90^\circ))$

Ratio of Shear Reinforcement $\rho_w = 314.16\text{mm}^2 / (200\text{mm} * 600\text{mm} * 1.0)$

Ratio of Shear Reinforcement $\rho_w = 0.002618$

Minimum Ratio of Shear Reinforcement $\rho_{w,min} = (0.08\sqrt{f_{ck}}) / f_{yk}$

Minimum Ratio of Shear Reinforcement $\rho_{w,min} = (0.08\sqrt{27\text{N/mm}^2}) / 500\text{N/mm}^2$

Minimum Ratio of Shear Reinforcement $\rho_{w,min} = 0.0008313$

Utilisation = $\rho_{w,min} / \rho_w = 0.0008313 / 0.002618 = 32\% \rightarrow \text{OK}$

Major Axis Shear - Top Steel

k factor = $1 + \sqrt{[200 / d_{top}]} \leq 2.0$

k factor = $1 + \sqrt{[200 / 392\text{mm}]} \leq 2.0$

k factor = $1.714 \leq 2.0 \rightarrow 1.714$

BS EN1992-1-1 6.2.2

Reinforcement ratio $\rho_l = A_{s,prov} / (B * d)$

Reinforcement ratio $\rho_l = 1206.4\text{mm}^2 / (600\text{mm} * 392\text{mm})$

Reinforcement ratio $\rho_l = 0.00513$

BS EN1992-1-1 6.2.2

$v_{min} = 0.035k^{3/2} * f_{ck}^{1/2}$

$v_{min} = 0.035(1.714)^{3/2} * (27\text{N/mm}^2)^{1/2}$

$v_{min} = 0.408$

UK NA to BS EN1992-1-1 6.2.2(1) and BS EN1992-1-1 Eq 6.3N

Shear Strength of Concrete $V_{Rd,c} = \max\{C_{Rd,c} * k * (100\rho_l * f_{ck})^{1/3}, v_{min}\} * B * d_{top}$

Shear Strength of Concrete $V_{Rd,c} = \max\{0.12 * 1.714 * (100 * 0.00513 * 27\text{N/mm}^2)^{1/3}, 0.408\} * 600\text{mm} * 392\text{mm}$

Shear Strength of Concrete $V_{Rd,c} = 116.2\text{kN}$

BS EN1992-1-1 Equation 6.2a and 6.2b

Cross Sectional Area of Shear Reinforcement $A_{sw} = (10\text{mm})^2 * \pi * (1/4) * 4 \text{ legs} = 314.16\text{mm}^2$

Strength reduction factor for concrete cracked in shear $v_1 = v = 0.6[1 - (f_{ck} / 250)]$

Strength reduction factor for concrete cracked in shear $v_1 = v = 0.6[1 - (27\text{N/mm}^2 / 250)]$

Strength reduction factor for concrete cracked in shear $v_1 = v = 0.5352$

BS EN1992-1-1 Equation 6.6N and the UK National Annex

Coefficient for state of stress in the compression chord $\alpha_{cw} = 1.0$ (no pre-stress applied)

BS EN1992-1-1 Section 6.2.3(3) Note 3

Coefficient for state of stress in the compression chord $\alpha_{cw} = 1.0$ (no pre-stress applied)
BS EN1992-1-1 Section 6.2.3(3) Note 3

Angle between the concrete strut and beam axis $\Theta = 21.8^\circ$ (see bottom steel for full equation)

Lever arm Z (from previous moment calculation) $Z = 372.4\text{mm}$
(if no moment is added use $Z = 0.9d$)

Design Strength of concrete in compression $f_{c wd} = \alpha_{ccw} * f_{ck} * (1 / \gamma_c)$
Design Strength of concrete in compression $f_{c wd} = 1.0 * 27\text{N/mm}^2 * (1 / 1.5)$
Design Strength of concrete in compression $f_{c wd} = 18\text{N/mm}^2$
 $\alpha_{ccw} = 1.0$ from UK National Annex to BS EN1992-1-1
BS EN1992-1-1 Equation 3.15

Maximum Shear Capacity of Beam $V_{Rd, max} = [\alpha_{cw} * B * Z * v_1 * f_{c wd}] / [\cot(45^\circ) + \tan(45^\circ)]$
Maximum Shear Capacity of Beam $V_{Rd, max} = [1.0 * 600\text{mm} * 372.4\text{mm} * 0.5352 * 18\text{N/mm}^2] / 2$
Maximum Shear Capacity of Beam $V_{Rd, max} = 1076.3\text{kN}$
BS EN1992-1-1 Equation 6.9
 $V_{y, Ed} < V_{Rd, max} \rightarrow 110\text{kN} < 1076.3\text{kN} \rightarrow$ Shear design is possible

Partial Material Factor for shear links $\gamma_s = 1.15$
BS EN1992-1-1 Table 2.1N

Design yield strength of shear reinforcement $f_{y wd} = f_{yk} / \gamma_s$
Design yield strength of shear reinforcement $f_{y wd} = 500\text{N/mm}^2 / 1.15$
Design yield strength of shear reinforcement $f_{y wd} = 434.8\text{N/mm}^2$

Capacity of current shear link arrangement $V_{Rd, s} = (A_{sw} / \text{link spacing}) * Z * f_{y wd} * \cot(\Theta)$
Capacity of current shear link arrangement $V_{Rd, s} = (314.16\text{mm}^2 / 200\text{mm}) * 372.4\text{mm} * 434.8\text{N/mm}^2 * \cot(21.8^\circ)$
Capacity of current shear link arrangement $V_{Rd, s} = 635.9\text{kN}$

Utilisation = $V_{y, Ed} / V_{Rd, s} = 110\text{kN} / 635.9\text{kN} = 17\% \rightarrow \text{OK}$

Ratio of Shear Reinforcement $\rho_w = A_{sw} / (\text{link spacing} * B * \sin(90^\circ))$
Ratio of Shear Reinforcement $\rho_w = 314.16\text{mm}^2 / (200\text{mm} * 600\text{mm} * 1.0)$
Ratio of Shear Reinforcement $\rho_w = 0.002618$

Minimum Ratio of Shear Reinforcement $\rho_{w, min} = (0.08\sqrt{f_{ck}}) / f_{yk}$
Minimum Ratio of Shear Reinforcement $\rho_{w, min} = (0.08\sqrt{27\text{N/mm}^2}) / 500\text{N/mm}^2$
Minimum Ratio of Shear Reinforcement $\rho_{w, min} = 0.0008313$

Utilisation = $\rho_{w, min} / \rho_w = 0.0008313 / 0.002618 = 32\% \rightarrow \text{OK}$

Minor Axis Shear - Side Left Steel

$$k \text{ factor} = 1 + \sqrt{[200 / d_{\text{sideL}}]} \leq 2.0$$

$$k \text{ factor} = 1 + \sqrt{[200 / 534\text{mm}]} \leq 2.0$$

$$k \text{ factor} = 1.612 \leq 2.0 \rightarrow 1.612$$

BS EN1992-1-1 6.2.2

$$\text{Reinforcement ratio } \rho_l = A_{s,\text{prov}} / (h * d)$$

$$\text{Reinforcement ratio } \rho_l = 452.4\text{mm}^2 / (450\text{mm} * 534\text{mm})$$

$$\text{Reinforcement ratio } \rho_l = 0.00188$$

BS EN1992-1-1 6.2.2

$$v_{\min} = 0.035k^{3/2} * f_{ck}^{1/2}$$

$$v_{\min} = 0.035(1.612)^{3/2} * (27\text{N/mm}^2)^{1/2}$$

$$v_{\min} = 0.372$$

UK NA to BS EN1992-1-1 6.2.2(1) and BS EN1992-1-1 Eq 6.3N

$$\text{Shear Strength of Concrete } V_{Rd,c} = \max\{C_{Rd,c} * k * (100\rho_l * f_{ck})^{1/3}, v_{\min}\} * h * d_{\text{sideL}}$$

$$\text{Shear Strength of Concrete } V_{Rd,c} = \max\{0.12 * 1.612 * (100 * 0.00118 * 27\text{N/mm}^2)^{1/3}, 0.372\} * 450\text{mm} * 534\text{mm}$$

$$\text{Shear Strength of Concrete } V_{Rd,c} = 89.4\text{kN}$$

BS EN1992-1-1 Equation 6.2a and 6.2b

$$\text{Cross Sectional Area of Shear Reinforcement } A_{sw} = (10\text{mm})^2 * \pi * (1/4) * 2 \text{ legs} = 157.08\text{mm}^2$$

$$\text{Strength reduction factor for concrete cracked in shear } v_1 = v = 0.6[1 - (f_{ck} / 250)]$$

$$\text{Strength reduction factor for concrete cracked in shear } v_1 = v = 0.6[1 - (27\text{N/mm}^2 / 250)]$$

$$\text{Strength reduction factor for concrete cracked in shear } v_1 = v = 0.5352$$

BS EN1992-1-1 Equation 6.6N and the UK National Annex

$$\text{Coefficient for state of stress in the compression chord } \alpha_{cw} = 1.0 \text{ (no pre-stress applied)}$$

BS EN1992-1-1 Section 6.2.3(3) Note 3

$$\text{Angle between the concrete strut and beam axis } \Theta = 21.8^\circ \text{ (see bottom steel for full equation)}$$

$$\text{Lever arm } Z \text{ (from previous moment calculation) } Z = 507.3\text{mm}$$

(if no moment is added use $Z = 0.9d$)

$$\text{Design Strength of concrete in compression } f_{c\text{wd}} = \alpha_{cw} * f_{ck} * (1 / \gamma_c)$$

$$\text{Design Strength of concrete in compression } f_{c\text{wd}} = 1.0 * 27\text{N/mm}^2 * (1 / 1.5)$$

$$\text{Design Strength of concrete in compression } f_{c\text{wd}} = 18\text{N/mm}^2$$

$$\alpha_{cw} = 1.0 \text{ from UK National Annex to BS EN1992-1-1}$$

BS EN1992-1-1 Equation 3.15

$$\text{Maximum Shear Capacity of Beam } V_{Rd,\text{max}} = [\alpha_{cw} * h * Z * v_1 * f_{c\text{wd}}] / [\cot(45^\circ) + \tan(45^\circ)]$$

$$\text{Maximum Shear Capacity of Beam } V_{Rd,\text{max}} = [1.0 * 450\text{mm} * 507.3\text{mm} * 0.5352 * 18\text{N/mm}^2] / 2$$

$$\text{Maximum Shear Capacity of Beam } V_{Rd,\text{max}} = 1099.6\text{kN}$$

BS EN1992-1-1 Equation 6.9

$$V_{z,\text{Ed}} < V_{Rd,\text{max}} \rightarrow 55\text{kN} < 1099.6\text{kN} \rightarrow \text{Shear design is possible}$$

Partial Material Factor for shear links $\gamma_s = 1.15$
BS EN1992-1-1 Table 2.1N

Design yield strength of shear reinforcement $f_{ywd} = f_{yk} / \gamma_s$

Design yield strength of shear reinforcement $f_{ywd} = 500\text{N/mm}^2 / 1.15$

Design yield strength of shear reinforcement $f_{ywd} = 434.8\text{N/mm}^2$

Capacity of current shear link arrangement $V_{Rd,s} = (A_{sw} / \text{link spacing}) * Z * f_{ywd} * \cot(\Theta)$

Capacity of current shear link arrangement $V_{Rd,s} = (157.08\text{mm}^2 / 200\text{mm}) * 507.3\text{mm} * 434.8\text{N/mm}^2 * \cot(21.8^\circ)$

Capacity of current shear link arrangement $V_{Rd,s} = 433.13\text{kN}$

Utilisation = $V_{z,Ed} / V_{Rd,s} = 55\text{kN} / 433.13\text{kN} = 13\% \rightarrow \text{OK}$

Ratio of Shear Reinforcement $\rho_w = A_{sw} / (\text{link spacing} * h * \sin(90^\circ))$

Ratio of Shear Reinforcement $\rho_w = 157.08\text{mm}^2 / (200\text{mm} * 450\text{mm} * 1.0)$

Ratio of Shear Reinforcement $\rho_w = 0.001745$

Minimum Ratio of Shear Reinforcement $\rho_{w,min} = (0.08 \sqrt{f_{ck}}) / f_{yk}$

Minimum Ratio of Shear Reinforcement $\rho_{w,min} = (0.08 \sqrt{27\text{N/mm}^2}) / 500\text{N/mm}^2$

Minimum Ratio of Shear Reinforcement $\rho_{w,min} = 0.0008313$

Utilisation = $\rho_{w,min} / \rho_w = 0.0008313 / 0.001745 = 48\% \rightarrow \text{OK}$

Minor Axis Shear - Side Right Steel

This is identical to the Side Left steel - see above

Check Crack Widths

Constant Values

Factor for bond between rebar and concrete $k_1 = 0.8$ (as we used high bond bars)

BS EN1992-1-1 Section 7.3.4

Factor for the Distribution of strain $k_2 = 0.5$ (because the applied forces are bending)

BS EN1992-1-1 Section 7.3.4

Factor $k_3 = 3.4$ (Recommended value from UK National Annex to BS EN1992-1-1 7.3.4(3))

Factor $k_4 = 0.425$ (Recommended value from UK National Annex to BS EN1992-1-1 7.3.4(3))

Mean Modulus of Elasticity of Concrete for Short Term Loading (i.e. @ time $t = 20$ days as specified by user)

$$E_{cm}(t) = E_{cm} * e^{0.3s[1 - \sqrt{\frac{28}{t}}]}$$

$$E_{cm}(t) = 28832\text{N/mm}^2 * e^{0.3*0.2[1 - \sqrt{\frac{28}{20\text{days}}}]}$$

The s in this formula is the cement factor and is a user input (refer to beginning of this document)

$$E_{cm}(t) = 28516.8\text{N/mm}^2$$

BS EN1992-1-1 Combining Equations 3.1, 3.2 and 3.5

Mean Modulus of Elasticity of Concrete for Long Term Loading (i.e. including creep effects)

$$E_{eff} = (1.05E_{cm}) / (1 + \phi_c)$$

$$E_{eff} = (28832\text{N/mm}^2) / (1 + 1.857)$$

$$E_{eff} = 10091.2\text{N/mm}^2$$

The ϕ_c factor is the creep factor and is a user input formula from BS EN1992-1-1 7.20

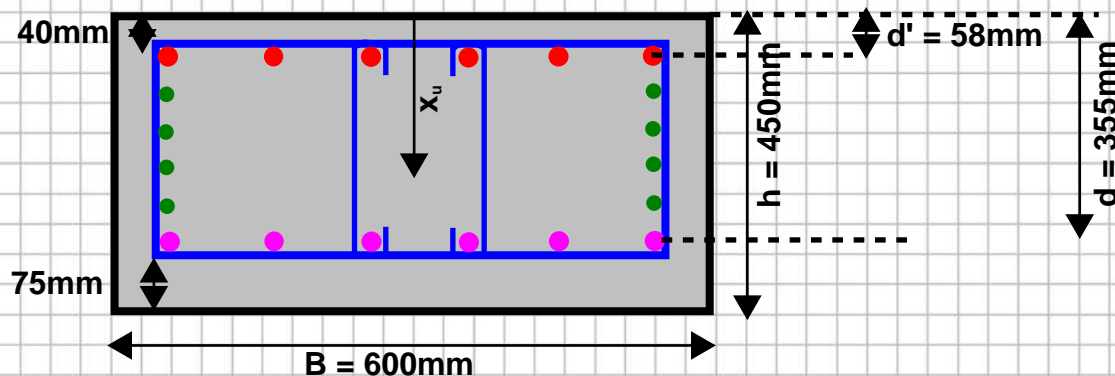
Modular Ratio for Short Term Loading $\alpha_e = E_s / E_{cm}(t) = 205000\text{N/mm}^2 / 28516.8\text{N/mm}^2 = 7.189$

Modular Ratio for Long Term Loading $\alpha_{e,creep} = E_s / E_{eff} = 205000\text{N/mm}^2 / 10091.2\text{N/mm}^2 = 20.314$

Uncracked Section Assessment - Short Term Loading

Modular Ratio $\alpha_e = 7.189$

Bottom Steel

Depth to Reinforcement in Tension $d = 355\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 40\text{mm top cover} + 10\text{mm link} + 16\text{mm top steel} / 2 = 58\text{mm}$ Provided Area of Reinforcement in Tension Zone $A_s = 6\text{H}20 = 1885\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A'_s = 6\text{H}16 = 1206.4\text{mm}^2$

Uncracked Neutral Axis Depth

$$x_u = \frac{\frac{h}{2}(Bh - A_s - A'_s) + \alpha_e A_s d + \alpha_e A'_s d'}{(Bh - A_s - A'_s) + \alpha_e A_s + \alpha_e A'_s}$$

See
Appendices
for derivation

$$x_u = 226.084\text{mm}$$

Uncracked Moment of Inertia

$$I_u = \frac{Bh^3}{12} + (Bh - A_s - A'_s)(x_u - \frac{h}{2})^2 + \alpha_e A_s (x_u - d)^2 + \alpha_e A'_s (x_u - d')^2$$

See
Appendices
for derivation

$$I_u = 5026802803\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{\text{SLS}} * (d - x_u) * (1 / I_u)$ Stress in the Concrete $\sigma_c = 35\text{kNm} * (355\text{mm} - 226.084\text{mm}) * (1 / 5026802803\text{mm}^4)$ Stress in the Concrete $\sigma_c = 0.898\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 0.898\text{N/mm}^2 * 7.189 = 6.455\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,\text{eff}} = (\min\{2.5[h - d], [h - x_u] / 3, h / 2\} * B) - A_s$ $A_{c,\text{eff}} = (\min\{2.5[450\text{mm} - 355\text{mm}], [450\text{mm} - 226.084\text{mm}] / 3, 450\text{mm} / 2\} * 600\text{mm}) - 1885\text{mm}^2$ $A_{c,\text{eff}} = 42898.2\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p,\text{eff}} = A_s / A_{c,\text{eff}} = 1885\text{mm}^2 / 42898.2\text{mm}^2 = 0.0439$

BS EN1992-1-1 Equation 7.10

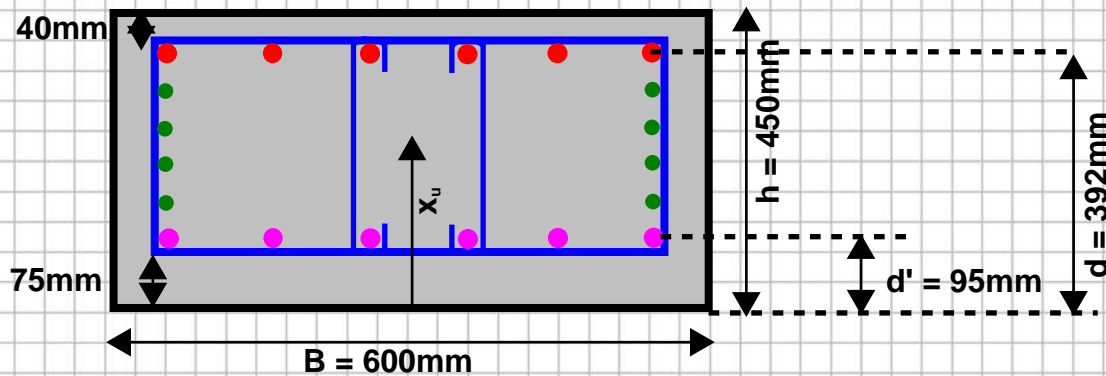
Maximum Crack Spacing $S_{r,\text{max}} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,\text{eff}}$ Maximum Crack Spacing $S_{r,\text{max}} = (3.4 * 75\text{mm}) + (0.8 * 0.5 * 0.425 * 20\text{mm}) / 0.0439$ Maximum Crack Spacing $S_{r,\text{max}} = 332.45\text{mm}$

BS EN1992-1-1 Equation 7.11

Uncracked Section Assessment - Short Term Loading

Modular Ratio $\alpha_e = 7.189$

Top Steel

Depth to Reinforcement in Tension $d = 392\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 75\text{mm}$ bottom cover + 10mm link + 20mm top steel / 2 = 95mmProvided Area of Reinforcement in Tension Zone $A_s = 6\text{H}16 = 1206.4\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A'_s = 6\text{H}20 = 1885\text{mm}^2$

Uncracked Neutral Axis Depth

$$x_u = \frac{\frac{h}{2}(Bh - A_s - A'_s) + \alpha_e A_s d + \alpha_e A'_s d'}{(Bh - A_s - A'_s) + \alpha_e A_s + \alpha_e A'_s}$$

See Appendices for derivation

$$x_u = 223.916\text{mm}$$

Uncracked Moment of Inertia

$$I_u = \frac{Bh^3}{12} + (Bh - A_s - A'_s)(x_u - \frac{h}{2})^2 + \alpha_e A_s (x_u - d)^2 + \alpha_e A'_s (x_u - d')^2$$

See Appendices for derivation

$$I_u = 5026802867\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{\text{SLS}} * (d - x_u) * (1 / I_u)$ Stress in the Concrete $\sigma_c = 15\text{kNm} * (392\text{mm} - 223.916\text{mm}) * (1 / 5026802867\text{mm}^4)$ Stress in the Concrete $\sigma_c = 0.502\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 0.502\text{N/mm}^2 * 7.189 = 3.609\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,\text{eff}} = (\min\{2.5[h - d], [h - x_u] / 3, h / 2\} * B) - A_s$ $A_{c,\text{eff}} = (\min\{2.5[450\text{mm} - 392\text{mm}], [450\text{mm} - 223.916\text{mm}] / 3, 450\text{mm} / 2\} * 600\text{mm}) - 1206.4\text{mm}^2$ $A_{c,\text{eff}} = 44010.4\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

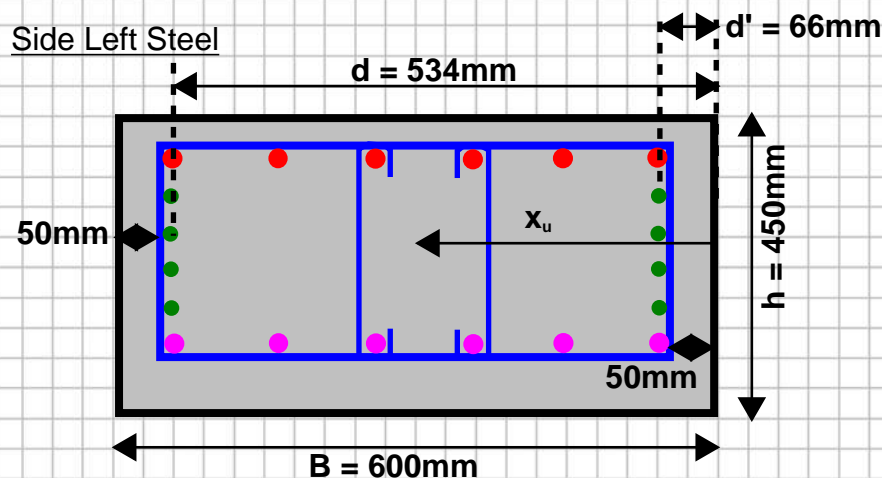
Ratio of Steel to effective area of concrete $p_{p,\text{eff}} = A_s / A_{c,\text{eff}} = 1206.4\text{mm}^2 / 44010.4\text{mm}^2 = 0.0274$

BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,\text{max}} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,\text{eff}}$ Maximum Crack Spacing $S_{r,\text{max}} = (3.4 * 40\text{mm}) + (0.8 * 0.5 * 0.425 * 16\text{mm}) / 0.0274$ Maximum Crack Spacing $S_{r,\text{max}} = 235.70\text{mm}$

BS EN1992-1-1 Equation 7.11

Uncracked Section Assessment - Short Term Loading

Modular Ratio $\alpha_e = 7.189$ Depth to Reinforcement in Tension $d = 534\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 50\text{mm}$ bottom cover + 10mm link + 12mm top steel / $2 = 66\text{mm}$ Provided Area of Reinforcement in Tension Zone $A_s = 4\text{H}12 = 452.4\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A'_s = 4\text{H}12 = 452.4\text{mm}^2$

Uncracked Neutral Axis Depth

$$x_u = \frac{\frac{B}{2}(hB - A_s - A'_s) + \alpha_e A_s d + \alpha_e A'_s d'}{(hB - A_s - A'_s) + \alpha_e A_s + \alpha_e A'_s}$$

See
Appendices
for derivation

$$x_u = 300.000\text{mm}$$

Uncracked Moment of Inertia

$$I_u = \frac{hB^3}{12} + (hB - A_s - A'_s)\left(x_u - \frac{B}{2}\right)^2 + \alpha_e A_s (x_u - d)^2 + \alpha_e A'_s (x_u - d')^2$$

See
Appendices
for derivation

$$I_u = 8456166272\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{\text{SLS}} * (d - x_u) * (1 / I_u)$ Stress in the Concrete $\sigma_c = 5\text{kNm} * (534\text{mm} - 300\text{mm}) * (1 / 8456166272\text{mm}^4)$ Stress in the Concrete $\sigma_c = 0.138\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 0.138\text{N/mm}^2 * 7.189 = 0.992\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,\text{eff}} = (\min\{2.5[B - d], [B - x_u] / 3, B / 2\} * h) - A_s$ $A_{c,\text{eff}} = (\min\{2.5[600\text{mm} - 534\text{mm}], [600\text{mm} - 300\text{mm}] / 3, 600\text{mm} / 2\} * 450\text{mm}) - 452.4\text{mm}^2$ $A_{c,\text{eff}} = 44547.6\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p,\text{eff}} = A_s / A_{c,\text{eff}} = 452.4\text{mm}^2 / 44547.6\text{mm}^2 = 0.01016$

BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,\text{max}} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,\text{eff}}$ Maximum Crack Spacing $S_{r,\text{max}} = (3.4 * 50\text{mm}) + (0.8 * 0.5 * 0.425 * 12\text{mm}) / 0.01016$ Maximum Crack Spacing $S_{r,\text{max}} = 370.787\text{mm}$

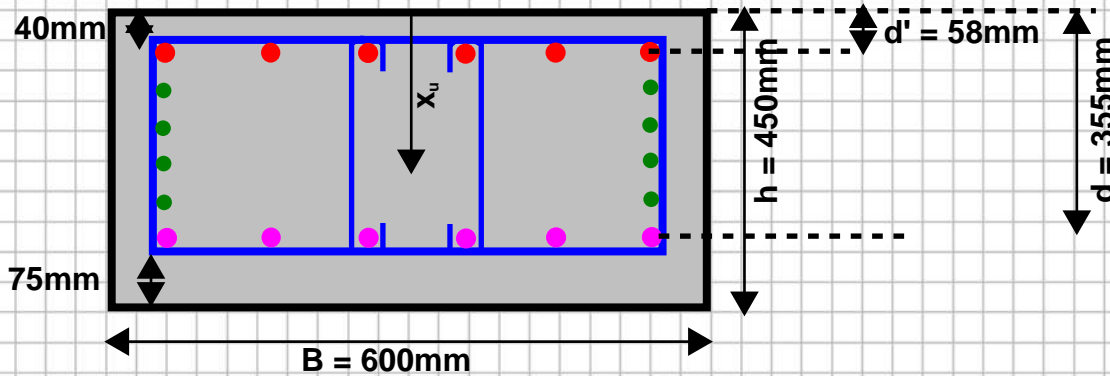
BS EN1992-1-1 Equation 7.11

The side right arrangement is identical to the side left arrangement

Uncracked Section Assessment - Long Term Loading

$$\text{Modular Ratio } \alpha_e = \alpha_{e, \text{creep}} = 20.314$$

Bottom Steel



Depth to Reinforcement in Tension $d = 355\text{mm}$ (from previous calculations)

Depth to Reinforcement in Compression $d' = 40\text{mm top cover} + 10\text{mm link} + 16\text{mm top steel} / 2 = 58\text{mm}$

Provided Area of Reinforcement in Tension Zone $A_s = 6\text{H}20 = 1885\text{mm}^2$

Provided Area of Reinforcement in Compression Zone $A'_s = 6\text{H}16 = 1206.4\text{mm}^2$

Uncracked Neutral Axis Depth

$$x_u = \frac{\frac{h}{2}(Bh - A_s - A'_s) + \alpha_e A_s d + \alpha_e A'_s d'}{(Bh - A_s - A'_s) + \alpha_e A_s + \alpha_e A'_s}$$

See Appendices for derivation

$$x_u = 227.683\text{mm}$$

Uncracked Moment of Inertia

$$I_u = \frac{Bh^3}{12} + (Bh - A_s - A'_s)(x_u - \frac{h}{2})^2 + \alpha_e A_s (x_u - d)^2 + \alpha_e A'_s (x_u - d')^2$$

See Appendices for derivation

$$I_u = 5884476183\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{\text{SLS}} * (d - x_u) * (1 / I_u)$

Stress in the Concrete $\sigma_c = 35\text{kNm} * (355\text{mm} - 227.683\text{mm}) * (1 / 5884476183\text{mm}^4)$

Stress in the Concrete $\sigma_c = 0.757\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 0.757\text{N/mm}^2 * 20.314 = 15.378\text{N/mm}^2$

Effective Area of Concrete in Tension $A_{c, \text{eff}} = (\min\{2.5[h - d], [h - x_u] / 3, h / 2\} * B) - A_s$

$A_{c, \text{eff}} = (\min\{2.5[450\text{mm} - 355\text{mm}], [450\text{mm} - 227.683\text{mm}] / 3, 450\text{mm} / 2\} * 600\text{mm}) - 1885\text{mm}^2$

$A_{c, \text{eff}} = 42578.4\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p, \text{eff}} = A_s / A_{c, \text{eff}} = 1885\text{mm}^2 / 42578.4\text{mm}^2 = 0.04427$

BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r, \text{max}} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p, \text{eff}}$

Maximum Crack Spacing $S_{r, \text{max}} = (3.4 * 75\text{mm}) + (0.8 * 0.5 * 0.425 * 20\text{mm}) / 0.04427$

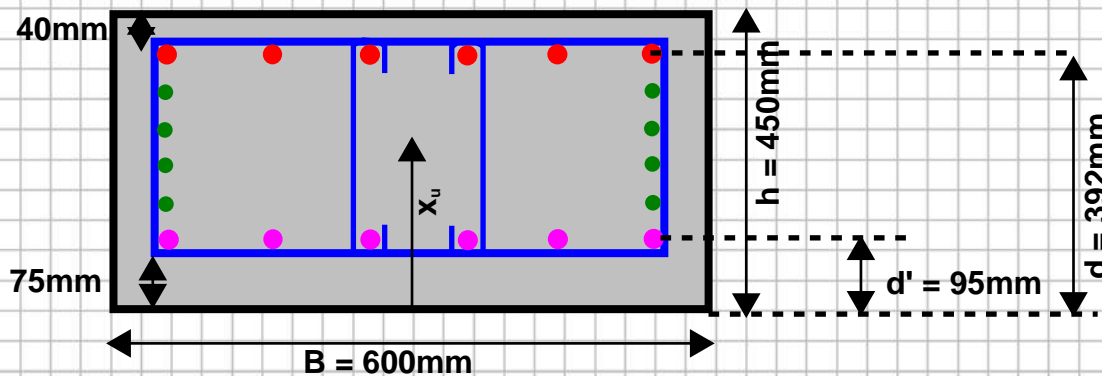
Maximum Crack Spacing $S_{r, \text{max}} = 331.8\text{mm}$

BS EN1992-1-1 Equation 7.11

Uncracked Section Assessment - Long Term Loading

Modular Ratio $\alpha_e = \alpha_{e,creep} = 20.314$

Top Steel

Depth to Reinforcement in Tension $d = 392\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 75\text{mm}$ bottom cover + 10mm link + 20mm top steel / 2 = 95mmProvided Area of Reinforcement in Tension Zone $A_s = 6\text{H}16 = 1206.4\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A'_s = 6\text{H}20 = 1885\text{mm}^2$

Uncracked Neutral Axis Depth

$$x_u = \frac{\frac{h}{2}(Bh - A_s - A'_s) + \alpha_e A_s d + \alpha_e A'_s d'}{(Bh - A_s - A'_s) + \alpha_e A_s + \alpha_e A'_s}$$

See
Appendices
for derivation

$$x_u = 222.315\text{mm}$$

Uncracked Moment of Inertia

$$I_u = \frac{Bh^3}{12} + (Bh - A_s - A'_s)(x_u - \frac{h}{2})^2 + \alpha_e A_s (x_u - d)^2 + \alpha_e A'_s (x_u - d')^2$$

See
Appendices
for derivation

$$I_u = 5884476181\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{SLS} * (d - x_u) * (1 / I_u)$ Stress in the Concrete $\sigma_c = 15\text{kNm} * (392\text{mm} - 222.315\text{mm}) * (1 / 5884476181\text{mm}^4)$ Stress in the Concrete $\sigma_c = 0.433\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 0.433\text{N/mm}^2 * 20.314 = 8.796\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,eff} = (\min\{2.5[h - d], [h - x_u] / 3, h / 2\} * B) - A_s$ $A_{c,eff} = (\min\{2.5[450\text{mm} - 392\text{mm}], [450\text{mm} - 222.315\text{mm}] / 3, 450\text{mm} / 2\} * 600\text{mm}) - 1206.4\text{mm}^2$ $A_{c,eff} = 44330.6\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p,eff} = A_s / A_{c,eff} = 1206.4\text{mm}^2 / 44330.6\text{mm}^2 = 0.02721$

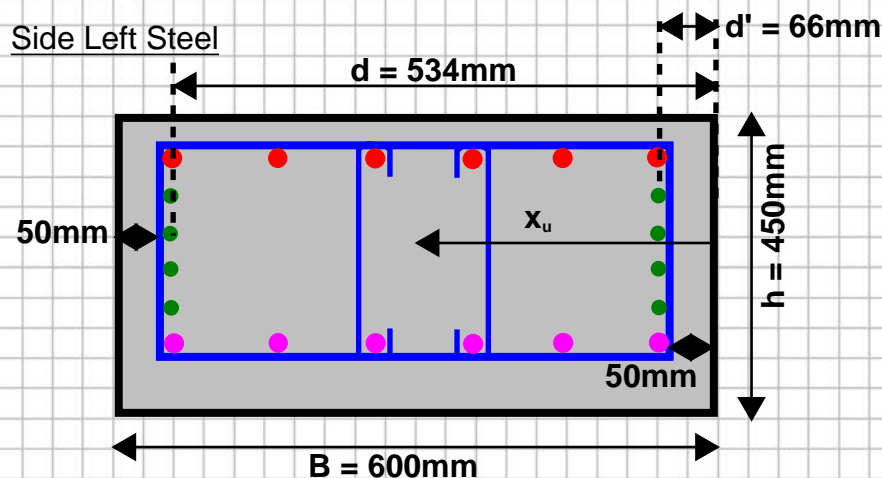
BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,max} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,eff}$ Maximum Crack Spacing $S_{r,max} = (3.4 * 40\text{mm}) + (0.8 * 0.5 * 0.425 * 16\text{mm}) / 0.02721$ Maximum Crack Spacing $S_{r,max} = 235.963\text{mm}$

BS EN1992-1-1 Equation 7.11

Modular Ratio $\alpha_e = \alpha_{e,creep} = 20.314$

Uncracked Section Assessment - Short Term Loading

Depth to Reinforcement in Tension $d = 534\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 50\text{mm}$ bottom cover + 10mm link + 12mm top steel / 2 = 66mm Provided Area of Reinforcement in Tension Zone $A_s = 4\text{H}12 = 452.4\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A'_s = 4\text{H}12 = 452.4\text{mm}^2$

Uncracked Neutral Axis Depth

$$x_u = \frac{\frac{B}{2}(hB - A_s - A'_s) + \alpha_e A_s d + \alpha_e A'_s d'}{(hB - A_s - A'_s) + \alpha_e A_s + \alpha_e A'_s}$$

See
Appendices
for derivation

$$x_u = 300\text{mm}$$

Uncracked Moment of Inertia

$$I_u = \frac{hB^3}{12} + (hB - A_s - A'_s)\left(x_u - \frac{B}{2}\right)^2 + \alpha_e A_s (x_u - d)^2 + \alpha_e A'_s (x_u - d')^2$$

See
Appendices
for derivation

$$I_u = 9106421150\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{SLS} * (d - x_u) * (1 / I_u)$ Stress in the Concrete $\sigma_c = 5\text{kNm} * (534\text{mm} - 300\text{mm}) * (1 / 9106421150\text{mm}^4)$ Stress in the Concrete $\sigma_c = 0.128\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 0.128\text{N/mm}^2 * 20.314 = 2.600\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,eff} = (\min\{2.5[B - d], [B - x_u] / 3, B / 2\} * h) - A_s$ $A_{c,eff} = (\min\{2.5[600\text{mm} - 534\text{mm}], [600\text{mm} - 300\text{mm}] / 3, 600\text{mm} / 2\} * 450\text{mm}) - 452.4\text{mm}^2$ $A_{c,eff} = 44547.6\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p,eff} = A_s / A_{c,eff} = 452.4\text{mm}^2 / 44547.6\text{mm}^2 = 0.01016$

BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,max} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,eff}$ Maximum Crack Spacing $S_{r,max} = (3.4 * 50\text{mm}) + (0.8 * 0.5 * 0.425 * 12\text{mm}) / 0.01016$ Maximum Crack Spacing $S_{r,max} = 370.787\text{mm}$

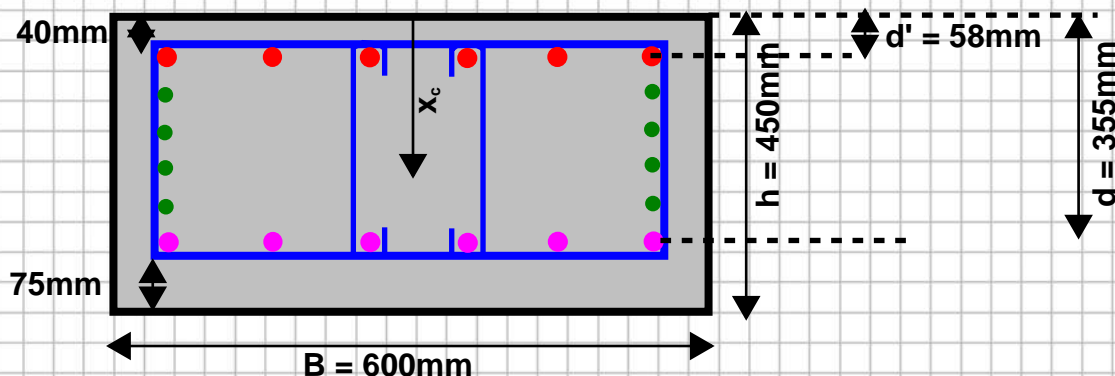
BS EN1992-1-1 Equation 7.11

The side right arrangement is identical to the side left arrangement

Cracked Section Assessment - Short Term Loading

Modular Ratio $\alpha_e = 7.189$

Bottom Steel

Depth to Reinforcement in Tension $d = 355\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 40\text{mm top cover} + 10\text{mm link} + 16\text{mm top steel} / 2 = 58\text{mm}$ Provided Area of Reinforcement in Tension Zone $A_s = 6\text{H}20 = 1885\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A'_s = 6\text{H}16 = 1206.4\text{mm}^2$

Cracked Neutral Axis Depth

$$0 = \underbrace{-\frac{B}{2}x_c^2}_{a} + \underbrace{\left(\frac{A'_s}{2} - \alpha_e(A_s + A'_s)\right)x_c}_{b} + \underbrace{\alpha_e(A_s d + A'_s d')}_{c} \quad : \text{ solve for } x_c \quad x_c = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x_c = 101.845\text{mm}$$

Cracked Moment of Inertia

$$I_c = \frac{Bx_c^3}{3} - \frac{x_c^2 A'_s}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A'_s (x_c - d')^2$$

$$I_c = 1093285724\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{\text{SLS}} * (d - x_c) * (1 / I_c)$ Stress in the Concrete $\sigma_c = 35\text{kNm} * (355\text{mm} - 101.845\text{mm}) * (1 / 1093285724\text{mm}^4)$ Stress in the Concrete $\sigma_c = 8.104\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 8.104\text{N/mm}^2 * 7.189 = 58.259\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,\text{eff}} = (\min\{2.5[h - d], [h - x_c] / 3, h / 2\} * B) - A_s$ $A_{c,\text{eff}} = (\min\{2.5[450\text{mm} - 355\text{mm}], [450\text{mm} - 101.845\text{mm}] / 3, 450\text{mm} / 2\} * 600\text{mm}) - 1885\text{mm}^2$ $A_{c,\text{eff}} = 67746\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p,\text{eff}} = A_s / A_{c,\text{eff}} = 1885\text{mm}^2 / 67746\text{mm}^2 = 0.02782$

BS EN1992-1-1 Equation 7.10

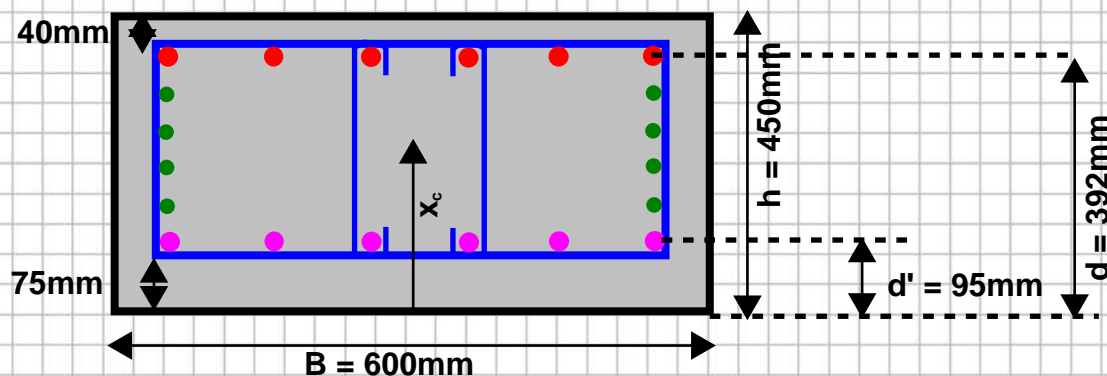
Maximum Crack Spacing $S_{r,\text{max}} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,\text{eff}}$ Maximum Crack Spacing $S_{r,\text{max}} = (3.4 * 75\text{mm}) + (0.8 * 0.5 * 0.425 * 20\text{mm}) / 0.02782$ Maximum Crack Spacing $S_{r,\text{max}} = 377.214\text{mm}$

BS EN1992-1-1 Equation 7.11

Cracked Section Assessment - Short Term Loading

Modular Ratio $\alpha_e = 7.189$

Top Steel

Depth to Reinforcement in Tension $d = 392\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 75\text{mm}$ bottom cover + 10mm link + 20mm top steel / $2 = 95\text{mm}$ Provided Area of Reinforcement in Tension Zone $A_s = 6\text{H}16 = 1206.4\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A_s' = 6\text{H}20 = 1885\text{mm}^2$

Cracked Neutral Axis Depth

$$0 = -\frac{B}{2}x_c^2 + \left(\frac{A_s'}{2} - \alpha_e(A_s + A_s')\right)x_c + \alpha_e(A_s d + A_s' d') \quad : \text{ solve for } x_c \quad x_c = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x_c = 94.461\text{mm}$$

Cracked Moment of Inertia

$$I_c = \frac{Bx_c^3}{3} - \frac{x_c^2 A_s'}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A_s' (x_c - d')^2$$

$$I_c = 932170993.3\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{\text{SLS}} * (d - x_c) * (1 / I_c)$ Stress in the Concrete $\sigma_c = 15\text{kNm} * (392\text{mm} - 94.461\text{mm}) * (1 / 932170993.3\text{mm}^4)$ Stress in the Concrete $\sigma_c = 4.788\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 4.788\text{N/mm}^2 * 7.189 = 34.421\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,\text{eff}} = (\min\{2.5[h - d], [h - x_c] / 3, h / 2\} * B) - A_s$ $A_{c,\text{eff}} = (\min\{2.5[450\text{mm} - 392\text{mm}], [450\text{mm} - 94.461\text{mm}] / 3, 450\text{mm} / 2\} * 600\text{mm}) - 1206.4\text{mm}^2$ $A_{c,\text{eff}} = 69901.4\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

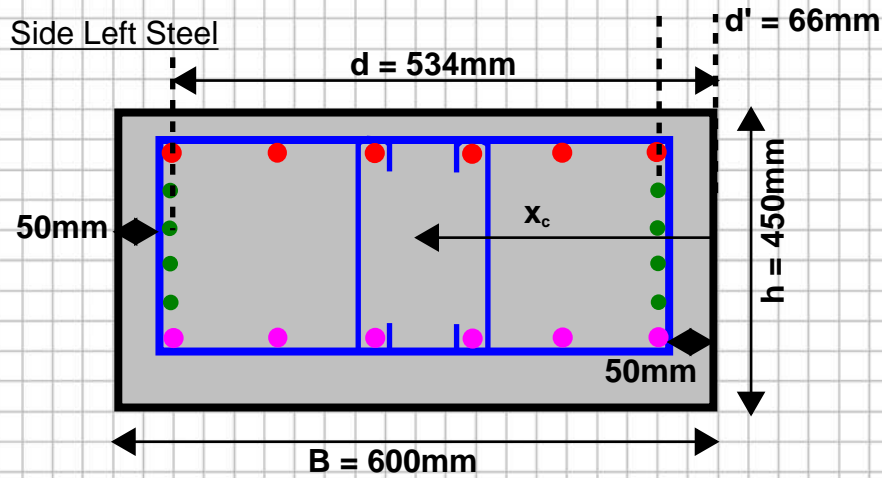
Ratio of Steel to effective area of concrete $p_{p,\text{eff}} = A_s / A_{c,\text{eff}} = 1206.4\text{mm}^2 / 69901.4\text{mm}^2 = 0.01726$

BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,\text{max}} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,\text{eff}}$ Maximum Crack Spacing $S_{r,\text{max}} = (3.4 * 40\text{mm}) + (0.8 * 0.5 * 0.425 * 16\text{mm}) / 0.01726$ Maximum Crack Spacing $S_{r,\text{max}} = 293.589\text{mm}$

BS EN1992-1-1 Equation 7.11

Cracked Section Assessment - Short Term Loading

Modular Ratio $\alpha_e = 7.189$ Depth to Reinforcement in Tension $d = 534\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 50\text{mm}$ bottom cover + 10mm link + 12mm top steel / 2 = 66mmProvided Area of Reinforcement in Tension Zone $A_s = 4\text{H}12 = 452.4\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A_s' = 4\text{H}12 = 452.4\text{mm}^2$

Cracked Neutral Axis Depth

$$0 = \underbrace{-\frac{h}{2}x_c^2}_{a} + \underbrace{\left(\frac{A_s'}{2} - \alpha_e(A_s + A_s')\right)x_c}_{b} + \underbrace{\alpha_e(A_s d + A_s' d')}_{c} \quad : \text{ solve for } x_c \quad x_c = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x_c = 80.215\text{mm}$$

Cracked Moment of Inertia

$$I_c = \frac{hx_c^3}{3} - \frac{x_c^2 A_s'}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A_s' (x_c - d')^2$$

$$I_c = 747067354.9\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{\text{SLS}} * (d - x_c) * (1 / I_c)$ Stress in the Concrete $\sigma_c = 5\text{kNm} * (534\text{mm} - 80.215\text{mm}) * (1 / 747067354.9\text{mm}^4)$ Stress in the Concrete $\sigma_c = 3.037\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 3.037\text{N/mm}^2 * 7.189 = 21.833\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,\text{eff}} = (\min\{2.5[B - d], [B - x_c] / 3, B / 2\} * h) - A_s$ $A_{c,\text{eff}} = (\min\{2.5[600\text{mm} - 534\text{mm}], [600\text{mm} - 80.215\text{mm}] / 3, 600\text{mm} / 2\} * 450\text{mm}) - 452.4\text{mm}^2$ $A_{c,\text{eff}} = 73798\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p,\text{eff}} = A_s / A_{c,\text{eff}} = 452.4\text{mm}^2 / 73798\text{mm}^2 = 0.006130$

BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,\text{max}} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,\text{eff}}$ Maximum Crack Spacing $S_{r,\text{max}} = (3.4 * 50\text{mm}) + (0.8 * 0.5 * 0.425 * 12\text{mm}) / 0.006130$ Maximum Crack Spacing $S_{r,\text{max}} = 502.789\text{mm}$

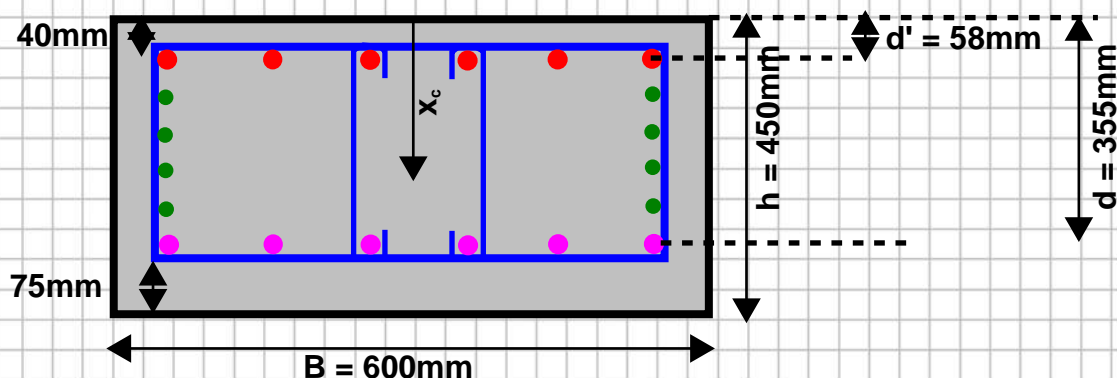
BS EN1992-1-1 Equation 7.11

The side right arrangement is identical to the side left arrangement

Modular Ratio $\alpha_e = \alpha_{e,creep} = 20.314$

Cracked Section Assessment - Long Term Loading

Bottom Steel

Depth to Reinforcement in Tension $d = 355\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 40\text{mm top cover} + 10\text{mm link} + 16\text{mm top steel} / 2 = 58\text{mm}$ Provided Area of Reinforcement in Tension Zone $A_s = 6\text{H}20 = 1885\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A_s' = 6\text{H}16 = 1206.4\text{mm}^2$

Cracked Neutral Axis Depth

$$0 = \underbrace{-\frac{B}{2}x_c^2}_{a} + \underbrace{\left(\frac{A_s'}{2} - \alpha_e(A_s + A_s')\right)x_c}_{b} + \underbrace{\alpha_e(A_s d + A_s' d')}_{c} \quad : \text{ solve for } x_c \quad x_c = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x_c = 142.908\text{mm}$$

Cracked Moment of Inertia

$$I_c = \frac{Bx_c^3}{3} - \frac{x_c^2 A_s'}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A_s' (x_c - d')^2$$

$$I_c = 2476717195\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{SLS} * (d - x_c) * (1 / I_c)$ Stress in the Concrete $\sigma_c = 35\text{kNm} * (355\text{mm} - 142.908\text{mm}) * (1 / 2476717195\text{mm}^4)$ Stress in the Concrete $\sigma_c = 2.997\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 2.997\text{N/mm}^2 * 20.314 = 60.881\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,eff} = (\min\{ 2.5[h - d] , [h - x_c] / 3 , h / 2 \} * B) - A_s$ $A_{c,eff} = (\min\{ 2.5[450\text{mm} - 355\text{mm}] , [450\text{mm} - 142.908\text{mm}] / 3 , 450\text{mm} / 2 \} * 600\text{mm}) - 1885\text{mm}^2$ $A_{c,eff} = 59533.4\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p,eff} = A_s / A_{c,eff} = 1885\text{mm}^2 / 59533.4\text{mm}^2 = 0.03166$

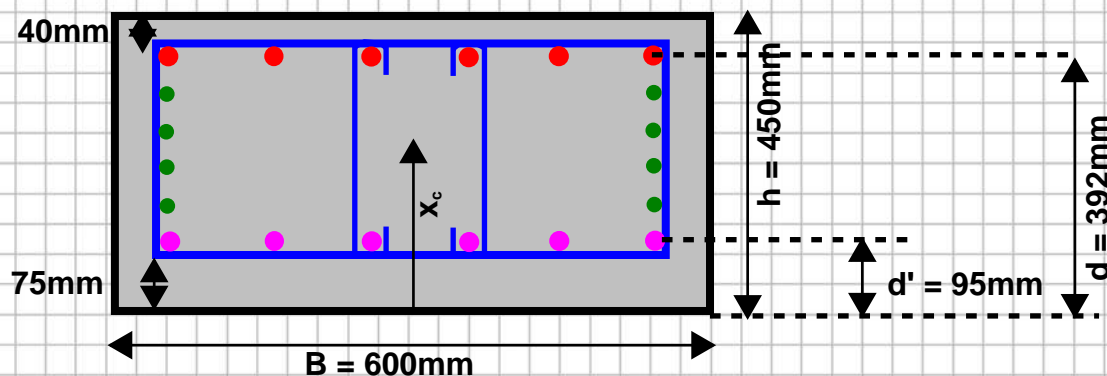
BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,max} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,eff}$ Maximum Crack Spacing $S_{r,max} = (3.4 * 75\text{mm}) + (0.8 * 0.5 * 0.425 * 20\text{mm}) / 0.03166$ Maximum Crack Spacing $S_{r,max} = 362.391\text{mm}$

BS EN1992-1-1 Equation 7.11

Cracked Section Assessment - Long Term Loading

Top Steel

Depth to Reinforcement in Tension $d = 392\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 75\text{mm}$ bottom cover + 10mm link + 20mm top steel / $2 = 95\text{mm}$ Provided Area of Reinforcement in Tension Zone $A_s = 6\text{H}16 = 1206.4\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A'_s = 6\text{H}20 = 1885\text{mm}^2$

Cracked Neutral Axis Depth

$$0 = \underbrace{-\frac{B}{2}x_c^2}_{a} + \underbrace{\left(\frac{A'_s}{2} - \alpha_e(A_s + A'_s)\right)x_c}_{b} + \underbrace{\alpha_e(A_s d + A'_s d')}_{c} \quad : \text{ solve for } x_c \quad x_c = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x_c = 130.950\text{mm}$$

Cracked Moment of Inertia

$$I_c = \frac{Bx_c^3}{3} - \frac{x_c^2 A'_s}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A'_s (x_c - d')^2$$

$$I_c = 2160579218\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{SLS} * (d - x_c) * (1 / I_c)$ Stress in the Concrete $\sigma_c = 15\text{kNm} * (392\text{mm} - 130.950\text{mm}) * (1 / 2160579218\text{mm}^4)$ Stress in the Concrete $\sigma_c = 1.812\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 1.812\text{N/mm}^2 * 20.314 = 36.809\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,eff} = (\min\{2.5[h - d], [h - x_c] / 3, h / 2\} * B) - A_s$ $A_{c,eff} = (\min\{2.5[450\text{mm} - 392\text{mm}], [450\text{mm} - 130.950\text{mm}] / 3, 450\text{mm} / 2\} * 600\text{mm}) - 1206.4\text{mm}^2$ $A_{c,eff} = 62603.6\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

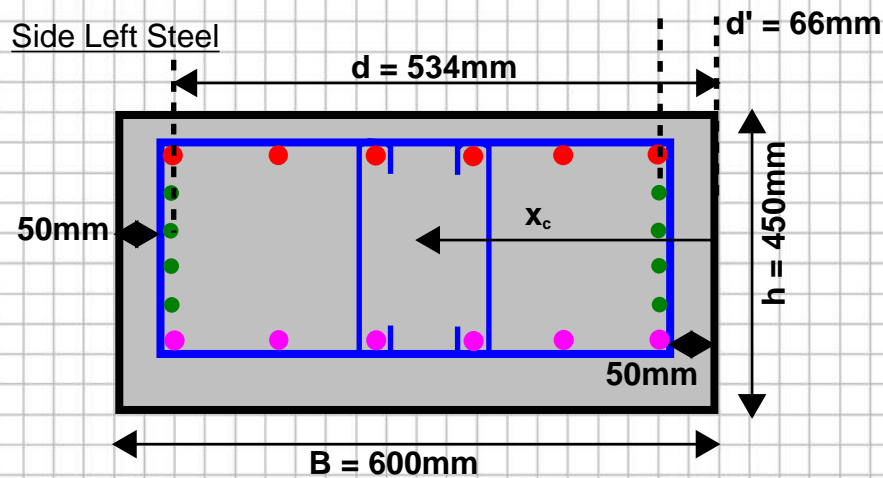
Ratio of Steel to effective area of concrete $p_{p,eff} = A_s / A_{c,eff} = 1206.4\text{mm}^2 / 62603.6\text{mm}^2 = 0.01927$

BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,max} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,eff}$ Maximum Crack Spacing $S_{r,max} = (3.4 * 40\text{mm}) + (0.8 * 0.5 * 0.425 * 16\text{mm}) / 0.01927$ Maximum Crack Spacing $S_{r,max} = 277.152\text{mm}$

BS EN1992-1-1 Equation 7.11

Cracked Section Assessment - Short Term Loading

Modular Ratio $\alpha_e = \alpha_{e,creep} = 20.314$ Depth to Reinforcement in Tension $d = 534\text{mm}$ (from previous calculations)Depth to Reinforcement in Compression $d' = 50\text{mm}$ bottom cover + 10mm link + 12mm top steel / 2 = 66mmProvided Area of Reinforcement in Tension Zone $A_s = 4\text{H}12 = 452.4\text{mm}^2$ Provided Area of Reinforcement in Compression Zone $A'_s = 4\text{H}12 = 452.4\text{mm}^2$

Cracked Neutral Axis Depth

$$0 = \underbrace{-\frac{h}{2}x_c^2}_{a} + \underbrace{\left(\frac{A'_s}{2} - \alpha_e(A_s + A'_s)\right)x_c}_{b} + \underbrace{\alpha_e(A_s d + A'_s d')}_{c} \quad : \text{ solve for } x_c \quad x_c = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x_c = 121.319\text{mm}$$

Cracked Moment of Inertia

$$I_c = \frac{hx_c^3}{3} - \frac{x_c^2 A'_s}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A'_s (x_c - d')^2$$

$$I_c = 1859417762\text{mm}^4$$

Stress in the Concrete $\sigma_c = M_{SLs} * (d - x_c) * (1 / I_c)$ Stress in the Concrete $\sigma_c = 5\text{kNm} * (534\text{mm} - 121.319\text{mm}) * (1 / 1859417762\text{mm}^4)$ Stress in the Concrete $\sigma_c = 1.110\text{N/mm}^2$

Engineers Bending Equation

Stress in the Steel $\sigma_s = \sigma_c * \alpha_e = 1.110\text{N/mm}^2 * 20.314 = 22.549\text{N/mm}^2$ Effective Area of Concrete in Tension $A_{c,eff} = (\min\{2.5[B - d], [B - x_c] / 3, B / 2\} * h) - A_s$ $A_{c,eff} = (\min\{2.5[600\text{mm} - 534\text{mm}], [600\text{mm} - 121.319\text{mm}] / 3, 600\text{mm} / 2\} * 450\text{mm}) - 452.4\text{mm}^2$ $A_{c,eff} = 71349.75\text{mm}^2$

BS EN1992-1-1 7.3.2(3)

Ratio of Steel to effective area of concrete $p_{p,eff} = A_s / A_{c,eff} = 452.4\text{mm}^2 / 71349.75\text{mm}^2 = 0.006341$

BS EN1992-1-1 Equation 7.10

Maximum Crack Spacing $S_{r,max} = (k_3 * \text{cover}) + (k_1 * k_2 * k_4 * \text{bar diameter}) / p_{p,eff}$ Maximum Crack Spacing $S_{r,max} = (3.4 * 50\text{mm}) + (0.8 * 0.5 * 0.425 * 12\text{mm}) / 0.006341$ Maximum Crack Spacing $S_{r,max} = 491.716\text{mm}$

BS EN1992-1-1 Equation 7.11

The side right arrangement is identical to the side left arrangement

Modular Ratio $\alpha_e = 7.189$

Short Term Loading - Crack Width Assessment

Bottom Steel

Mean Concrete Tensile Strength at t=20 days

$$f_{ct,eff} = f_{ctm(t)} = f_{ctm} * e^{\alpha_s(1 - \sqrt{\frac{28}{t}})}$$

 $\alpha = 1.0$ BS EN1992-1-1 Eq 3.14

$$f_{ct,eff} = f_{ctm(t)} = 2.7 \text{ N/mm}^2 * e^{1.0 * 0.2(1 - \sqrt{\frac{28}{20 \text{ days}}})}$$

s = 0.2 factor for cement type - user input

$$f_{ct,eff} = 2.603 \text{ N/mm}^2$$

BS EN1992-1-1 Combining Eq 3.2 & 3.4

$$\text{Cracking Moment } M_c = (f_{ct,eff} * I_u) / (d - x_u)$$

$$\text{Cracking Moment } M_c = (2.603 \text{ N/mm}^2 * 5026802803 \text{ mm}^4) / (355 \text{ mm} - 226.084 \text{ mm})$$

$$\text{Cracking Moment } M_c = 101.498 \text{ kNm (engineers bending equation)}$$

Is the applied moment greater than the cracking moment? (i.e. is $\sigma_c \geq f_{ct,eff}$)

35kNm applied moment < 101.498kNm therefore section is uncracked

Maximum Crack spacing $S_{r,max} = 332.45 \text{ mm}$ (from previous assessment)Factor for short duration of the load $k_t = 0.6$

BS EN1992-1-1 Section 7.3.4(2)

Mean Strain in Rebar - Mean Strain in Concrete

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{p_{p,eff}} (1 + \alpha_e p_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \quad \text{BS EN1992-1-1 Equation 7.9}$$

$$\epsilon_{sm} - \epsilon_{cm} = 0.00001889$$

$$\text{Crack width } w_k = S_{r,max} * (\epsilon_{sm} - \epsilon_{cm})$$

$$\text{Crack width } w_k = 332.45 \text{ mm} * (0.00001889)$$

BS EN1992-1-1 Equation 7.8

$$\text{Crack width } w_k = 0.00628 \text{ mm}$$

Limiting crack width = 0.3mm

Utilisation = $0.00628 \text{ mm} / 0.3 \text{ mm} = 2.09\%$ --> OK

Short Term Loading - Crack Width Assessment

Modular Ratio $\alpha_e = 7.189$

Top Steel

Mean Concrete Tensile Strength at t=20 days

$$f_{ct,eff} = f_{ctm(t)} = f_{ctm} * e^{\alpha_s(1 - \sqrt{\frac{28}{t}})}$$

 $\alpha = 1.0$ BS EN1992-1-1 Eq 3.14

$$f_{ct,eff} = f_{ctm(t)} = 2.7 \text{ N/mm}^2 * e^{1.0 * 0.2(1 - \sqrt{\frac{28}{20 \text{ days}}})}$$

s = 0.2 factor for cement type - user input

$$f_{ct,eff} = 2.603 \text{ N/mm}^2$$

BS EN1992-1-1 Combining Eq 3.2 & 3.4

$$\text{Cracking Moment } M_c = (f_{ct,eff} * I_u) / (d - x_u)$$

$$\text{Cracking Moment } M_c = (2.603 \text{ N/mm}^2 * 5026802867 \text{ mm}^4) / (392 \text{ mm} - 223.916 \text{ mm})$$

$$\text{Cracking Moment } M_c = 77.846 \text{ kNm (engineers bending equation)}$$

Is the applied moment greater than the cracking moment? (i.e. is $\sigma_c \geq f_{ct,eff}$)

15kNm applied moment < 77.846kNm therefore section is uncracked

Maximum Crack spacing $S_{r,max} = 235.70 \text{ mm}$ (from previous assessment)Factor for short duration of the load $k_t = 0.6$

BS EN1992-1-1 Section 7.3.4(2)

Mean Strain in Rebar - Mean Strain in Concrete

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \quad \text{BS EN1992-1-1 Equation 7.9}$$

$$\epsilon_{sm} - \epsilon_{cm} = 0.00001056$$

$$\text{Crack width } w_k = S_{r,max} * (\epsilon_{sm} - \epsilon_{cm})$$

$$\text{Crack width } w_k = 235.70 \text{ mm} * (0.00001056)$$

BS EN1992-1-1 Equation 7.8

$$\text{Crack width } w_k = 0.00249 \text{ mm}$$

Limiting crack width = 0.3mm

Utilisation = $0.00249 \text{ mm} / 0.3 \text{ mm} = 0.83\%$ --> OK

Short Term Loading - Crack Width Assessment

Modular Ratio $\alpha_e = 7.189$

Side Left Steel

Mean Concrete Tensile Strength at t=20 days

$$f_{ct,eff} = f_{ctm(t)} = f_{ctm} * e^{\alpha_s(1 - \sqrt{\frac{28}{t}})}$$

 $\alpha = 1.0$ BS EN1992-1-1 Eq 3.14

$$f_{ct,eff} = f_{ctm(t)} = 2.7 \text{ N/mm}^2 * e^{1.0 * 0.2(1 - \sqrt{\frac{28}{20 \text{ days}}})}$$

s = 0.2 factor for cement type - user input

$$f_{ct,eff} = 2.603 \text{ N/mm}^2$$

BS EN1992-1-1 Combining Eq 3.2 & 3.4

$$\text{Cracking Moment } M_c = (f_{ct,eff} * I_u) / (d - x_u)$$

$$\text{Cracking Moment } M_c = (2.603 \text{ N/mm}^2 * 8456166272 \text{ mm}^4) / (534 \text{ mm} - 300 \text{ mm})$$

$$\text{Cracking Moment } M_c = 94.066 \text{ kNm (engineers bending equation)}$$

Is the applied moment greater than the cracking moment? (i.e. is $\sigma_c \geq f_{ct,eff}$)

5kNm applied moment < 94.066kNm therefore section is uncracked

Maximum Crack spacing $S_{r,max} = 370.787 \text{ mm}$ (from previous assessment)Factor for short duration of the load $k_t = 0.6$

BS EN1992-1-1 Section 7.3.4(2)

Mean Strain in Rebar - Mean Strain in Concrete

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \quad \text{BS EN1992-1-1 Equation 7.9}$$

$$\epsilon_{sm} - \epsilon_{cm} = 0.00000290$$

$$\text{Crack width } w_k = S_{r,max} * (\epsilon_{sm} - \epsilon_{cm})$$

$$\text{Crack width } w_k = 370.787 \text{ mm} * (0.00000290)$$

BS EN1992-1-1 Equation 7.8

$$\text{Crack width } w_k = 0.001075 \text{ mm}$$

Limiting crack width = 0.3mm

Utilisation = 0.001075mm / 0.3mm = 0.36% --> OK

The side right arrangement is identical to the side left arrangement

Long Term Loading - Crack Width Assessment

$$\text{Modular Ratio } \alpha_e = \alpha_{e, \text{creep}} = 20.314$$

Bottom Steel

Mean Concrete Tensile Strength at t=28 days

$$f_{ct, \text{eff}} = f_{ctm} = 2.700 \text{ N/mm}^2$$

BS EN1992-1-1 Table 3.1

$$\text{Cracking Moment } M_c = (f_{ct, \text{eff}} * I_u) / (d - x_u)$$

$$\text{Cracking Moment } M_c = (2.700 \text{ N/mm}^2 * 5884476183 \text{ mm}^4) / (355 \text{ mm} - 227.683 \text{ mm})$$

$$\text{Cracking Moment } M_c = 124.792 \text{ kNm (engineers bending equation)}$$

Is the applied moment greater than the cracking moment? (i.e. is $\sigma_c \geq f_{ct, \text{eff}}$)

35kNm applied moment < 124.792kNm therefore section is uncracked

Maximum Crack spacing $S_{r, \text{max}} = 331.8 \text{ mm}$ (from previous assessment)Factor for long duration of the load $k_t = 0.4$

BS EN1992-1-1 Section 7.3.4(2)

Mean Strain in Rebar - Mean Strain in Concrete

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct, \text{eff}}}{\rho_{p, \text{eff}}} (1 + \alpha_e \rho_{p, \text{eff}})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \quad \text{BS EN1992-1-1 Equation 7.9}$$

$$\epsilon_{sm} - \epsilon_{cm} = 0.00004501$$

$$\text{Crack width } w_k = S_{r, \text{max}} * (\epsilon_{sm} - \epsilon_{cm})$$

$$\text{Crack width } w_k = 331.8 \text{ mm} * (0.00004501)$$

BS EN1992-1-1 Equation 7.8

$$\text{Crack width } w_k = 0.0149 \text{ mm}$$

Limiting crack width = 0.3mm

Utilisation = 0.0149mm / 0.3mm = 4.97% --> OK

Long Term Loading - Crack Width Assessment

$$\text{Modular Ratio } \alpha_e = \alpha_{e, \text{creep}} = 20.314$$

Top Steel

Mean Concrete Tensile Strength at t=28 days

$$f_{ct, \text{eff}} = f_{ctm} = 2.700 \text{ N/mm}^2$$

BS EN1992-1-1 Table 3.1

$$\text{Cracking Moment } M_c = (f_{ct, \text{eff}} * I_u) / (d - x_u)$$

$$\text{Cracking Moment } M_c = (2.700 \text{ N/mm}^2 * 5884476181 \text{ mm}^4) / (392 \text{ mm} - 222.315 \text{ mm})$$

$$\text{Cracking Moment } M_c = 93.632 \text{ kNm (engineers bending equation)}$$

Is the applied moment greater than the cracking moment? (i.e. is $\sigma_c \geq f_{ct, \text{eff}}$)

15kNm applied moment < 93.632kNm therefore section is uncracked

Maximum Crack spacing $S_{r, \text{max}} = 235.963 \text{ mm}$ (from previous assessment)Factor for long duration of the load $k_t = 0.4$

BS EN1992-1-1 Section 7.3.4(2)

Mean Strain in Rebar - Mean Strain in Concrete

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct, \text{eff}}}{\rho_{p, \text{eff}}} (1 + \alpha_e \rho_{p, \text{eff}})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \quad \text{BS EN1992-1-1 Equation 7.9}$$

$$\epsilon_{sm} - \epsilon_{cm} = 0.00002574$$

$$\text{Crack width } w_k = S_{r, \text{max}} * (\epsilon_{sm} - \epsilon_{cm})$$

$$\text{Crack width } w_k = 235.963 \text{ mm} * (0.00002574)$$

BS EN1992-1-1 Equation 7.8

$$\text{Crack width } w_k = 0.006074 \text{ mm}$$

Limiting crack width = 0.3mm

Utilisation = 0.006074mm / 0.3mm = 2.02% --> OK

Long Term Loading - Crack Width Assessment

$$\text{Modular Ratio } \alpha_e = \alpha_{e, \text{creep}} = 20.314$$

Side Left Steel

Mean Concrete Tensile Strength at t=28 days

$$f_{ct, \text{eff}} = f_{ctm} = 2.700 \text{ N/mm}^2$$

BS EN1992-1-1 Table 3.1

$$\text{Cracking Moment } M_c = (f_{ct, \text{eff}} * I_u) / (d - x_u)$$

$$\text{Cracking Moment } M_c = (2.700 \text{ N/mm}^2 * 9106421150 \text{ mm}^4) / (534 \text{ mm} - 300 \text{ mm})$$

$$\text{Cracking Moment } M_c = 105.074 \text{ kNm (engineers bending equation)}$$

Is the applied moment greater than the cracking moment? (i.e. is $\sigma_c \geq f_{ct, \text{eff}}$)

5kNm applied moment < 105.074kNm therefore section is uncracked

Maximum Crack spacing $S_{r, \text{max}} = 370.787 \text{ mm}$ (from previous assessment)Factor for long duration of the load $k_t = 0.4$

BS EN1992-1-1 Section 7.3.4(2)

Mean Strain in Rebar - Mean Strain in Concrete

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct, \text{eff}}}{\rho_{p, \text{eff}}} (1 + \alpha_e \rho_{p, \text{eff}})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \quad \text{BS EN1992-1-1 Equation 7.9}$$

$$\epsilon_{sm} - \epsilon_{cm} = 0.00000761$$

$$\text{Crack width } w_k = S_{r, \text{max}} * (\epsilon_{sm} - \epsilon_{cm})$$

$$\text{Crack width } w_k = 370.787 \text{ mm} * (0.00000761)$$

BS EN1992-1-1 Equation 7.8

$$\text{Crack width } w_k = 0.002822 \text{ mm}$$

Limiting crack width = 0.3mm

Utilisation = $0.002822 \text{ mm} / 0.3 \text{ mm} = 0.941\% \rightarrow \text{OK}$

The side right arrangement is identical to the side left arrangement

Short Term Loading - Minimum Area of Reinforcement to Control Cracking

Modular Ratio $\alpha_e = 7.189$

Bottom Steel

$$\text{Area of Concrete within Tension Zone } A_{ct} = (h - x_u) * B = (450\text{mm} - 226.084\text{mm}) * 600\text{mm} = 134349.6\text{mm}^2$$

Coefficient for self equilibrating stresses k:

if $h \leq 300\text{mm} \rightarrow k = 1$ if $h \geq 800\text{mm} \rightarrow k = 0.65$ if $300\text{mm} < h < 800\text{mm}$ then use linear interpolation

$$k = 1 - \{[(450\text{mm} - 300\text{mm}) / (800\text{mm} - 300\text{mm})] * (1 - 0.65)\}$$

$$k = 0.895$$

BS EN1992-1-1 Section 7.3.2

Coefficient for stress distribution $k_c = 0.4$ (simplified - no axial force applied)

BS EN1992-1-1 Equation 7.2

$$\text{Minimum Area of Reinforcement } A_{s,min} = k_c * k * f_{ct,eff} * A_{ct} * (1 / f_{yk})$$

$$\text{Minimum Area of Reinforcement } A_{s,min} = 0.4 * 0.895 * 2.603\text{N/mm}^2 * 134349.6\text{mm}^2 * (1 / 500\text{N/mm}^2)$$

$$\text{Minimum Area of Reinforcement } A_{s,min} = 250.393\text{mm}^2$$

BS EN1992-1-1 Equation 7.1

$$\text{Area of Steel Provided } A_{s,prov} = 6\text{H}20 = 1885\text{mm}^2$$

$$\text{Utilisation} = A_{s,min} / A_{s,prov} = 250.393\text{mm}^2 / 1885\text{mm}^2 = 13.2\% \rightarrow \text{OK}$$

Short Term Loading - Minimum Area of Reinforcement to Control Cracking

Modular Ratio $\alpha_e = 7.189$

Top Steel

$$\text{Area of Concrete within Tension Zone } A_{ct} = (h - x_u) * B = (450\text{mm} - 223.916\text{mm}) * 600\text{mm} = 135650.4\text{mm}^2$$

Coefficient for self equilibrating stresses k:

if $h \leq 300\text{mm} \rightarrow k = 1$ if $h \geq 800\text{mm} \rightarrow k = 0.65$ if $300\text{mm} < h < 800\text{mm}$ then use linear interpolation

$$k = 1 - \{[(450\text{mm} - 300\text{mm}) / (800\text{mm} - 300\text{mm})] * (1 - 0.65)\}$$

$$k = 0.895$$

BS EN1992-1-1 Section 7.3.2

Coefficient for stress distribution $k_c = 0.4$ (simplified - no axial force applied)

BS EN1992-1-1 Equation 7.2

$$\text{Minimum Area of Reinforcement } A_{s,min} = k_c * k * f_{ct,eff} * A_{ct} * (1 / f_{yk})$$

$$\text{Minimum Area of Reinforcement } A_{s,min} = 0.4 * 0.895 * 2.603\text{N/mm}^2 * 135650.4\text{mm}^2 * (1 / 500\text{N/mm}^2)$$

$$\text{Minimum Area of Reinforcement } A_{s,min} = 252.818\text{mm}^2$$

BS EN1992-1-1 Equation 7.1

$$\text{Area of Steel Provided } A_{s,prov} = 6\text{H}16 = 1206.4\text{mm}^2$$

$$\text{Utilisation} = A_{s,min} / A_{s,prov} = 252.818\text{mm}^2 / 1206.4\text{mm}^2 = 20.9\% \rightarrow \text{OK}$$

Short Term Loading - Minimum Area of Reinforcement to Control Cracking

Modular Ratio $\alpha_e = 7.189$

Side Left Steel

Area of Concrete within Tension Zone $A_{ct} = (B - x_u) * h = (600\text{mm} - 300\text{mm}) * 450\text{mm} = 135000\text{mm}^2$

Coefficient for self equilibrating stresses k:

if $B \leq 300\text{mm} \rightarrow k = 1$

if $B \geq 800\text{mm} \rightarrow k = 0.65$

if $300\text{mm} < B < 800\text{mm}$ then use linear interpolation

$k = 1 - \{[(600\text{mm} - 300\text{mm}) / (800\text{mm} - 300\text{mm})] * (1 - 0.65)\}$

$k = 0.790$

BS EN1992-1-1 Section 7.3.2

Coefficient for stress distribution $k_c = 0.4$ (simplified - no axial force applied)

BS EN1992-1-1 Equation 7.2

Minimum Area of Reinforcement $A_{s,min} = k_c * k * f_{ct,eff} * A_{ct} * (1 / f_{yk})$

Minimum Area of Reinforcement $A_{s,min} = 0.4 * 0.790 * 2.603\text{N/mm}^2 * 135000\text{mm}^2 * (1 / 500\text{N/mm}^2)$

Minimum Area of Reinforcement $A_{s,min} = 222.088\text{mm}^2$

BS EN1992-1-1 Equation 7.1

Area of Steel Provided $A_{s,prov} = 4\text{H}12 = 452.4\text{mm}^2$

Utilisation = $A_{s,min} / A_{s,prov} = 222.088\text{mm}^2 / 452.4\text{mm}^2 = 49\% \rightarrow \text{OK}$

The side right arrangement is identical to the side left arrangement

Long Term Loading - Minimum Area of Reinforcement to Control Cracking

Modular Ratio $\alpha_e = 20.314$

Bottom Steel

Area of Concrete within Tension Zone $A_{ct} = (h - x_u) * B = (450\text{mm} - 227.683\text{mm}) * 600\text{mm} = 133390.2\text{mm}^2$

Coefficient for self equilibrating stresses k:

if $h \leq 300\text{mm} \rightarrow k = 1$

if $h \geq 800\text{mm} \rightarrow k = 0.65$

if $300\text{mm} < h < 800\text{mm}$ then use linear interpolation

$k = 1 - \{[(450\text{mm} - 300\text{mm}) / (800\text{mm} - 300\text{mm})] * (1 - 0.65)\}$

$k = 0.895$

BS EN1992-1-1 Section 7.3.2

Coefficient for stress distribution $k_c = 0.4$ (simplified - no axial force applied)

BS EN1992-1-1 Equation 7.2

Minimum Area of Reinforcement $A_{s,min} = k_c * k * f_{ct,eff} * A_{ct} * (1 / f_{yk})$

Minimum Area of Reinforcement $A_{s,min} = 0.4 * 0.895 * 2.700\text{N/mm}^2 * 133390.2\text{mm}^2 * (1 / 500\text{N/mm}^2)$

Minimum Area of Reinforcement $A_{s,min} = 257.870\text{mm}^2$

BS EN1992-1-1 Equation 7.1

Area of Steel Provided $A_{s,prov} = 6\text{H}20 = 1885\text{mm}^2$

Utilisation = $A_{s,min} / A_{s,prov} = 257.870\text{mm}^2 / 1885\text{mm}^2 = 13.7\% \rightarrow \text{OK}$

Long Term Loading - Minimum Area of Reinforcement to Control Cracking

Modular Ratio $\alpha_e = 20.314$

Top Steel

$$\text{Area of Concrete within Tension Zone } A_{ct} = (h - x_u) * B = (450\text{mm} - 222.315\text{mm}) * 600\text{mm} = 136611\text{mm}^2$$

Coefficient for self equilibrating stresses k:

$$\text{if } h \leq 300\text{mm} \rightarrow k = 1$$

$$\text{if } h \geq 800\text{mm} \rightarrow k = 0.65$$

if $300\text{mm} < h < 800\text{mm}$ then use linear interpolation

$$k = 1 - \{[(450\text{mm} - 300\text{mm}) / (800\text{mm} - 300\text{mm})] * (1 - 0.65)\}$$

$$k = 0.895$$

BS EN1992-1-1 Section 7.3.2

Coefficient for stress distribution $k_c = 0.4$ (simplified - no axial force applied)

BS EN1992-1-1 Equation 7.2

$$\text{Minimum Area of Reinforcement } A_{s,min} = k_c * k * f_{ct,eff} * A_{ct} * (1 / f_{yk})$$

$$\text{Minimum Area of Reinforcement } A_{s,min} = 0.4 * 0.895 * 2.700\text{N/mm}^2 * 136611\text{mm}^2 * (1 / 500\text{N/mm}^2)$$

$$\text{Minimum Area of Reinforcement } A_{s,min} = 264.096\text{mm}^2$$

BS EN1992-1-1 Equation 7.1

$$\text{Area of Steel Provided } A_{s,prov} = 6\text{H16} = 1206.4\text{mm}^2$$

$$\text{Utilisation} = A_{s,min} / A_{s,prov} = 264.096\text{mm}^2 / 1206.4\text{mm}^2 = 21.9\% \rightarrow \text{OK}$$

Long Term Loading - Minimum Area of Reinforcement to Control Cracking

Modular Ratio $\alpha_e = 20.314$

Side Left Steel

$$\text{Area of Concrete within Tension Zone } A_{ct} = (B - x_u) * h = (600\text{mm} - 300\text{mm}) * 450\text{mm} = 135000\text{mm}^2$$

Coefficient for self equilibrating stresses k:

$$\text{if } B \leq 300\text{mm} \rightarrow k = 1$$

$$\text{if } B \geq 800\text{mm} \rightarrow k = 0.65$$

if $300\text{mm} < B < 800\text{mm}$ then use linear interpolation

$$k = 1 - \{[(600\text{mm} - 300\text{mm}) / (800\text{mm} - 300\text{mm})] * (1 - 0.65)\}$$

$$k = 0.790$$

BS EN1992-1-1 Section 7.3.2

Coefficient for stress distribution $k_c = 0.4$ (simplified - no axial force applied)

BS EN1992-1-1 Equation 7.2

$$\text{Minimum Area of Reinforcement } A_{s,min} = k_c * k * f_{ct,eff} * A_{ct} * (1 / f_{yk})$$

$$\text{Minimum Area of Reinforcement } A_{s,min} = 0.4 * 0.790 * 2.700\text{N/mm}^2 * 135000\text{mm}^2 * (1 / 500\text{N/mm}^2)$$

$$\text{Minimum Area of Reinforcement } A_{s,min} = 230.364\text{mm}^2$$

BS EN1992-1-1 Equation 7.1

$$\text{Area of Steel Provided } A_{s,prov} = 4\text{H12} = 452.4\text{mm}^2$$

$$\text{Utilisation} = A_{s,min} / A_{s,prov} = 230.364\text{mm}^2 / 452.4\text{mm}^2 = 50.9\% \rightarrow \text{OK}$$

The side right arrangement is identical to the side left arrangement

Check Torsion

Geometry

Overall cross sectional area of the beam $A = B * h = 600\text{mm} * 450\text{mm} = 270000\text{mm}^2$

Overall perimeter of the beam $u = B + B + h + h = 600\text{mm} + 600\text{mm} + 450\text{mm} + 450\text{mm} = 2100\text{mm}$

Distance from the edge of the beam to the centreline of the longitudinal reinforcement:

Bottom Steel $d_{\text{bottom}}' = 75\text{mm cover} + 10\text{mm link} + 20\text{mm bar diameter} / 2 = 95\text{mm}$

Top Steel $d_{\text{top}}' = 40\text{mm cover} + 10\text{mm link} + 16\text{mm bar diameter} / 2 = 58\text{mm}$

Side Left Steel $d_{\text{sideleft}}' = 50\text{mm cover} + 10\text{mm link} + 12\text{mm bar diameter} / 2 = 66\text{mm}$

Side Right Steel $d_{\text{sideright}}' = 50\text{mm cover} + 10\text{mm link} + 12\text{mm bar diameter} / 2 = 66\text{mm}$

Effective Wall Thickness:

Bottom Steel $t_{\text{ef,bottom}} = \max\{A / u, 2 * d_{\text{bottom}}'\} \leq 2 * h$

Bottom Steel $t_{\text{ef,bottom}} = \max\{270000\text{mm}^2 / 2100\text{mm}, 2 * 95\text{mm}\} \leq 2 * 450\text{mm}$

Bottom Steel $t_{\text{ef,bottom}} = 190\text{mm}$

Top Steel $t_{\text{ef,top}} = \max\{A / u, 2 * d_{\text{top}}'\} \leq 2 * h$

Top Steel $t_{\text{ef,top}} = \max\{270000\text{mm}^2 / 2100\text{mm}, 2 * 58\text{mm}\} \leq 2 * 450\text{mm}$

Top Steel $t_{\text{ef,top}} = 128.571\text{mm}$

Side Left Steel $t_{\text{ef,sideleft}} = \max\{A / u, 2 * d_{\text{sideleft}}'\} \leq 2 * B$

Side Left Steel $t_{\text{ef,sideleft}} = \max\{270000\text{mm}^2 / 2100\text{mm}, 2 * 66\text{mm}\} \leq 2 * 600\text{mm}$

Side Left Steel $t_{\text{ef,sideleft}} = 132\text{mm}$

Side Right Steel $t_{\text{ef,sideright}} = \max\{A / u, 2 * d_{\text{sideright}}'\} \leq 2 * B$

Side Right Steel $t_{\text{ef,sideright}} = \max\{270000\text{mm}^2 / 2100\text{mm}, 2 * 66\text{mm}\} \leq 2 * 600\text{mm}$

Side Right Steel $t_{\text{ef,sideright}} = 132\text{mm}$

BS EN1992-1-1 Section 6.3.2

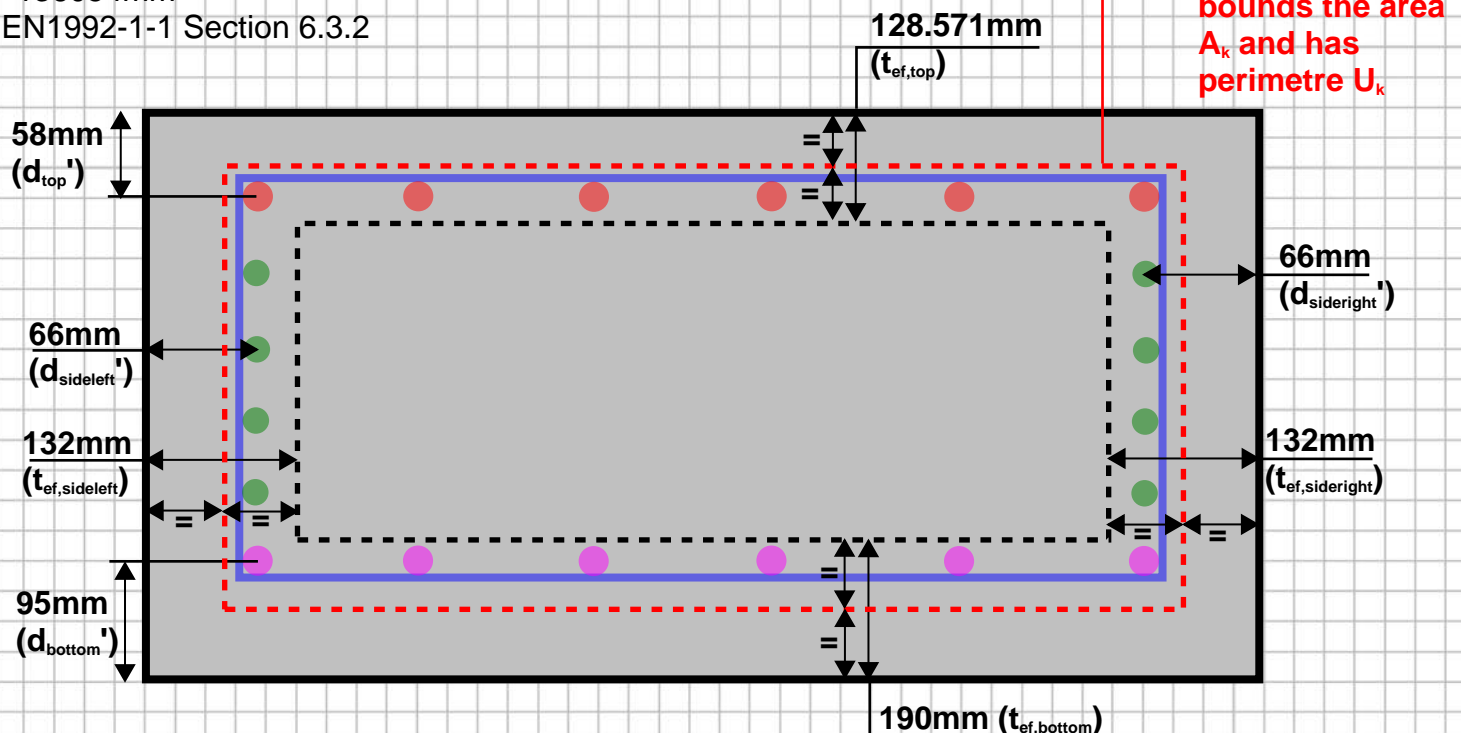
Cross sectional area enclosed by the centrelines of the connecting walls

$A_k = (h - [t_{\text{ef,bottom}} / 2] - [t_{\text{ef,top}} / 2]) * (B - [t_{\text{ef,sideleft}} / 2] - [t_{\text{ef,sideright}} / 2])$

$A_k = (450\text{mm} - [190\text{mm} / 2] - [128.571\text{mm} / 2]) * (600\text{mm} - [132\text{mm} / 2] - [132\text{mm} / 2])$

$A_k = 136054\text{mm}^2$

BS EN1992-1-1 Section 6.3.2



Check TorsionGeometry Continued

Perimeter of the area enclosed by centrelines of connecting walls

$$U_k = \{ 2 * (h - [t_{ef,bottom} / 2] - [t_{ef,top} / 2]) \} + \{ 2 * (B - [t_{ef,sidleft} / 2] - [t_{ef,sideright} / 2]) \}$$

$$U_k = \{ 2 * (450mm - [190mm / 2] - [128.571mm / 2]) \} + \{ 2 * (600mm - [132mm / 2] - [132mm / 2]) \}$$

$$U_k = 1517.42mm$$

BS EN1992-1-1 Section 6.3.2 and also refer to diagram on previous page

Total Applied Shear Stress

Applied Torsional shear stress in the "walls"

$$\tau_{t,Ed,bottom} = T_{Ed} / (2 * A_k * t_{ef,bottom}) = 7kNm / (2 * 136054mm^2 * 190mm) = 0.135N/mm^2$$

$$\tau_{t,Ed,top} = T_{Ed} / (2 * A_k * t_{ef,top}) = 7kNm / (2 * 136054mm^2 * 128.571mm) = 0.200N/mm^2$$

$$\tau_{t,Ed,sidleft} = T_{Ed} / (2 * A_k * t_{ef,sidleft}) = 7kNm / (2 * 136054mm^2 * 132mm) = 0.195N/mm^2$$

$$\tau_{t,Ed,sideright} = T_{Ed} / (2 * A_k * t_{ef,sideright}) = 7kNm / (2 * 136054mm^2 * 132mm) = 0.195N/mm^2$$

BS EN1992-1-1 Equation 6.26

Applied Shear Stress from Major Axis Shear:

$$v_{y,Ed,bottom} = V_{y,Ed} / (B * d) = 110kN / (600mm * 355mm) = 0.516N/mm^2$$

$$v_{y,Ed,top} = V_{y,Ed} / (B * d) = 110kN / (600mm * 392mm) = 0.468N/mm^2$$

Applied Shear Stress from Minor Axis Shear:

$$v_{z,Ed,sidleft} = V_{z,Ed} / (B * d) = 55kN / (450mm * 534mm) = 0.229N/mm^2$$

$$v_{z,Ed,sideright} = V_{z,Ed} / (B * d) = 55kN / (450mm * 534mm) = 0.229N/mm^2$$

Maximum Torsion Capacity

$$\text{Strength reduction factor for concrete cracked in shear } v_1 = v = 0.6[1 - (f_{ck} / 250)]$$

$$\text{Strength reduction factor for concrete cracked in shear } v_1 = v = 0.6[1 - (27N/mm^2 / 250)] = 0.5352$$

BS EN1992-1-1 Equation 6.6N and UK National Annex

Coefficient for State of Stress in Compression Chord $\alpha_{cw} = 1.0$

no prestress BS EN1992-1-1 6.2.3 Note 3

Partial Material Factor for Concrete $\gamma_c = 1.5$

BS EN1992-1-1 Table 2.1N

$$\text{Design Strength of Concrete in Compression } f_{cd} = \alpha_{cc} * f_{ck} * (1 / \gamma_c) = 0.85 * 27N/mm^2 * (1 / 1.5) = 15.3N/mm^2$$

BS EN1992-1-1 Equation 3.15 and UK National Annex ($\alpha_{cc} = 0.85$)

$$\text{Design Strength of Concrete in Compression } f_{c wd} = \alpha_{ccw} * f_{ck} * (1 / \gamma_c) = 1.0 * 27N/mm^2 * (1 / 1.5) = 18N/mm^2$$

BS EN1992-1-1 Equation 3.15 and UK National Annex ($\alpha_{ccw} = 1.0$)

Angle between concrete strut and beam axis $\Theta_t = 21.8^\circ$ (in all cases)

$$\theta_t = \min(\max[\frac{1}{2} \sin^{-1}(\min[\frac{2v_{t,Ed}}{(0.9 + \tau_{t,Ed})\alpha_{cw}f_{c wd}v_1}, 1]), 21.8^\circ], 45^\circ)$$

Check TorsionMaximum Torsion Capacity Continued

$$\text{Maximum Torsional Capacity of Beam } T_{Rd,max} = 2 * v_1 * \alpha_{cw} * f_{cd} * A_k * t_{ef} * \sin(\Theta_t) * \cos(\Theta_t)$$

$$\text{Bottom steel} = 2 * v_1 * \alpha_{cw} * f_{cd} * A_k * t_{ef,bottom} * \sin(\Theta_t) * \cos(\Theta_t)$$

$$\text{Bottom steel} = 2 * 0.5352 * 1.0 * 15.3\text{N/mm}^2 * 136054\text{mm}^2 * 190\text{mm} * \sin(21.8^\circ) * \cos(21.8^\circ) = 145.98\text{kNm}$$

$$\text{Top steel} = 2 * v_1 * \alpha_{cw} * f_{cd} * A_k * t_{ef,top} * \sin(\Theta_t) * \cos(\Theta_t)$$

$$\text{Top steel} = 2 * 0.5352 * 1.0 * 15.3\text{N/mm}^2 * 136054\text{mm}^2 * 128.571\text{mm} * \sin(21.8^\circ) * \cos(21.8^\circ) = 98.78\text{kNm}$$

$$\text{SideLeft Steel} = 2 * v_1 * \alpha_{cw} * f_{cd} * A_k * t_{ef,sideleft} * \sin(\Theta_t) * \cos(\Theta_t)$$

$$\text{SideLeft Steel} = 2 * 0.5352 * 1.0 * 15.3\text{N/mm}^2 * 136054\text{mm}^2 * 132\text{mm} * \sin(21.8^\circ) * \cos(21.8^\circ) = 101.42\text{kNm}$$

BS EN1992-1-1 Equation 6.30

Maximum Shear Capacity of Beam $V_{Rd,max}$

$$\text{Major Axis - Bottom Steel} = \alpha_{cw} * B * 0.9d * v_1 * f_{cwd} / (\cot(\Theta_t) + \tan(\Theta_t))$$

$$\text{Major Axis - Bottom Steel} = (1.0 * 600\text{mm} * 0.9 * 355\text{mm} * 0.5352 * 18\text{N/mm}^2) / (\cot(21.8^\circ) + \tan(21.8^\circ))$$

$$\text{Major Axis - Bottom Steel} = 636.78\text{kN}$$

$$\text{Major Axis - Top Steel} = \alpha_{cw} * B * 0.9d * v_1 * f_{cwd} / (\cot(\Theta_t) + \tan(\Theta_t))$$

$$\text{Major Axis - Top Steel} = (1.0 * 600\text{mm} * 0.9 * 392\text{mm} * 0.5352 * 18\text{N/mm}^2) / (\cot(21.8^\circ) + \tan(21.8^\circ))$$

$$\text{Major Axis - Top Steel} = 703.2\text{kN}$$

$$\text{Minor Axis - Side Left Steel} = \alpha_{cw} * h * 0.9d * v_1 * f_{cwd} / (\cot(\Theta_t) + \tan(\Theta_t))$$

$$\text{Minor Axis - Side Left Steel} = (1.0 * 450\text{mm} * 0.9 * 534\text{mm} * 0.5352 * 18\text{N/mm}^2) / (\cot(21.8^\circ) + \tan(21.8^\circ))$$

$$\text{Minor Axis - Side Left Steel} = 718.4\text{kN}$$

$$\text{Minor Axis - Side Right Steel} = 718.4\text{kN} \text{ (identical to side left steel)}$$

BS EN1992-1-1 Equation 6.9

Check interaction using the minimum capacities available

$$(T_{Ed} / T_{Rd,max}) + (V_{y,Ed} / V_{y,Rd,max}) + (V_{z,Ed} / V_{z,Rd,max})$$

$$(7\text{kNm} / 98.78\text{kNm}) + (110\text{kN} / 636.78\text{kN}) + (55\text{kN} / 718.4\text{kN}) = 32\% \rightarrow \text{Torsion design is possible}$$

Check Provided ReinforcementCharacteristic Axial Strength of Concrete (5% Fractile)

$$f_{ctk,0.05} = 0.7 * f_{ctm} = 0.7 * 2.70\text{N/mm}^2$$

$$f_{ctk,0.05} = 1.89\text{N/mm}^2$$

BS EN1992-1-1 Table 3.1

$$\text{Design axial strength of concrete } f_{ctd} = \alpha_{ct} * f_{ctk,0.05} * (1 / \gamma_c) = 1.0 * 1.89\text{N/mm}^2 * (1 / 1.5) = 1.26\text{N/mm}^2$$

BS EN1992-1-1 Equation 3.16 and UK National Annex

Maximum Torsional Capacity of the Beam with Shear Links

$$T_{Rd,c,bottom} = 2 * A_k * f_{ctd} * t_{ef,bottom} = 2 * 136054\text{mm}^2 * 1.26\text{N/mm}^2 * 190\text{mm} = 65.142\text{kNm}$$

$$T_{Rd,c,top} = 2 * A_k * f_{ctd} * t_{ef,top} = 2 * 136054\text{mm}^2 * 1.26\text{N/mm}^2 * 128.571\text{mm} = 44.081\text{kNm}$$

$$T_{Rd,c,sideleft} = 2 * A_k * f_{ctd} * t_{ef,sideleft} = 2 * 136054\text{mm}^2 * 1.26\text{N/mm}^2 * 132\text{mm} = 45.257\text{kNm}$$

$$T_{Rd,c,sideright} = 2 * A_k * f_{ctd} * t_{ef,sideright} = 2 * 136054\text{mm}^2 * 1.26\text{N/mm}^2 * 132\text{mm} = 45.257\text{kNm}$$

BS EN1992-1-1 Equation 6.28

Shear strength of concrete (previously calculated in shear design)

$$V_{Rd,c,bottom} = 128.83\text{kN}$$

$$V_{Rd,c,top} = 116.2\text{kN}$$

$$V_{Rd,c,sidleft} = 89.2\text{kN}$$

$$V_{Rd,c,sideright} = 89.2\text{kN}$$

Interaction Formula using worst case capacities (i.e. is torsion reinforcement required)

$$(T_{Ed} / T_{Rd,c}) + (V_{y,Ed} / V_{y,Rd,c}) + (V_{z,Ed} / V_{z,Rd,c})$$

$(7\text{kNm} / 44.081\text{kNm}) + (110\text{kN} / 116.2\text{kN}) + (55\text{kN} / 89.2\text{kN}) = 1.722 \rightarrow \text{FAIL}$ therefore provide torsion reinforcement

Partial Material Factor for reinforcement $\gamma_s = 1.15$

BS EN1992-1-1 Table 2.1N

Design strength of reinforcing steel $f_{yd} = f_{yk} / \gamma_s = 500\text{N/mm}^2 / 1.15 = 434.783\text{N/mm}^2$

Required area of additional reinforcement

$$A_{s,add,req} = (T_{Ed} * U_k * \cot(\Theta_t)) / (2 * A_k * f_{yd})$$

$$A_{s,add,req} = (7\text{kNm} * 1517.42\text{mm} * \cot(21.8^\circ)) / (2 * 136054\text{mm}^2 * 434.783\text{N/mm}^2) = 224.47\text{mm}^2$$

BS EN1992-1-1 Eq 6.28

Use 4H10 bars placed on 4 corners of the beam (314mm^2)

Required additional shear reinforcement for torsion

$$A_{s,add,link} = T_{Ed} / (2 * A_k * f_{yd} * \cot(\Theta_t))$$

$$A_{s,add,link} = 7\text{kNm} / (2 * 136054\text{mm}^2 * 434.832\text{N/mm}^2 * \cot(21.8^\circ))$$

$$A_{s,add,link} = 23.66\text{mm}^2/\text{m}$$

Maximum Spacing of Additional Links

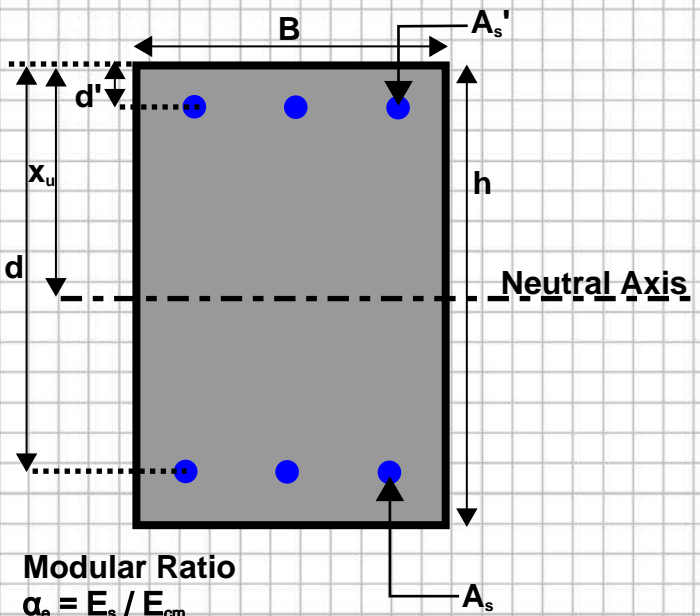
$$S_{w,max} = \min\{U_k / 8, B, h\}$$

$$S_{w,max} = \min\{1517.42\text{mm} / 8, 600\text{mm}, 450\text{mm}\} = 189.67\text{mm} \rightarrow \text{round to } 190\text{mm}$$

Possible size of additional links = H5@175mm

$$\text{Area provided} = ([5\text{mm}]^2 * \pi * 0.25) * (1000\text{mm} / 175\text{mm}) = 112.2\text{mm}^2/\text{m}'$$

Provide **Closed links** if possible

Uncracked Neutral Axis and Second Moment of Area Derivation**Cross Section Geometry****Input Variables:**

h = Height of the concrete beam/slab (mm)
 B = width of the concrete beam/slab (mm)
 A_s = Area of steel in bottom (mm²)
 A_s' = Area of steel in top of beam (mm²)
 d = Depth to bottom steel (mm)
 d' = Depth to top steel (mm)
 E_s = Modulus of elasticity of steel (N/mm²)
 (usually 205000N/mm²)
 E_{cm} = Mean Modulus of elasticity of concrete
 (N/mm²) (See BS EN1992-1-1 Table 3.1)

Solve for x_u neutral axis depth

Calculation Overview

The distance to the neutral axis is found by assuming that both the concrete above and below the neutral axis and the steel above and below the neutral axis contribute to the stiffness of the section. From here we convert all the steel elements into equivalent concrete elements using the modular ratio and we use the process of geometrical decomposition to find the neutral axis (i.e. $x_c = \sum (A * y) / \sum (A)$ where A are the cross sectional areas of each subdivided element and y is the distance from the top of the beam down to the centroid of each individual element).

Finding Distance to the Neutral Axis (x_u)

Sub Divided Element	Area (A)	Distance to Centroid (y)
Concrete Beam	$(B * h) - A_s - A_s'$	$h / 2$
Bottom Steel	$A_s * \alpha_e$	d
Top Steel	$A_s' * \alpha_e$	d'

$$x_u = \sum (A * y) / \sum (A)$$

$$x_u = \frac{\frac{h}{2}(Bh - A_s - A_s') + \alpha_e A_s d + \alpha_e A_s' d'}{(Bh - A_s - A_s') + \alpha_e A_s + \alpha_e A_s'}$$

This formula is simple to apply, just plug in the numbers and you'll get the uncracked neutral axis depth.

Finding Second Moment of Area

The second moment of area is found by applying the parallel axis theorem which is as follows:

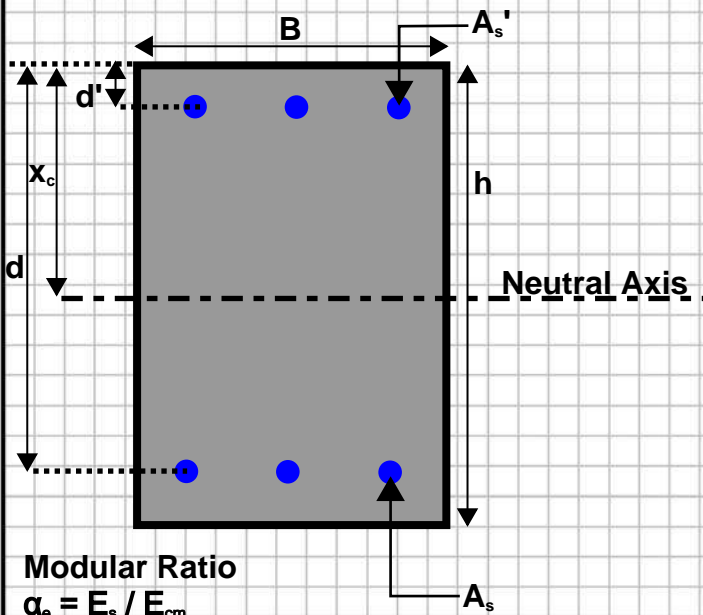
$$I_{total} = \sum (I) + \sum (A * y_{bar}^2)$$

where:

- I is the second moment of area for each individual element about it's centroid (for a rectangle it would be $bd^3/12$)
- A is the cross sectional area of each subdivided element
- y_{bar} is the distance from the neutral axis to the centroid of each subdivided element

Sub Divided Element	I	A	y _{bar}	A * y _{bar} ²
Concrete Beam	1/12 * B * h ³	(B * h) - A _s - A _s '	x _u - h/2	[(B * h) - A _s - A _s '] * [x _u - h/2] ²
Bottom Steel	negligible = 0	α _e * A _s	x _u - d	α _e A _s (x _u - d) ²
Top Steel	negligible = 0	α _e * A _s '	x _u - d'	α _e A _s '(x _u - d') ²

$$I_u = \frac{Bh^3}{12} + (Bh - A_s - A'_s)(x_u - \frac{h}{2})^2 + \alpha_e A_s (x_u - d)^2 + \alpha_e A'_s (x_u - d')^2$$

Cracked Neutral Axis and Second Moment of Area Derivation**Cross Section Geometry****Input Variables:**

h = Height of the concrete beam/slab (mm)
 B = width of the concrete beam/slab (mm)
 A_s = Area of steel in bottom (mm²)
 A_s' = Area of steel in top of beam (mm²)
 d = Depth to bottom steel (mm)
 d' = Depth to top steel (mm)
 E_s = Modulus of elasticity of steel (N/mm²)
 (usually 205000N/mm²)
 E_{cm} = Mean Modulus of elasticity of concrete
 (N/mm²) (See BS EN1992-1-1 Table 3.1)

Solve for x_c neutral axis depth

Calculation Overview

The distance to the neutral axis is found by assuming that all the concrete below the neutral axis provides nothing in terms of stiffness to the beam i.e. it is cracked and essentially worthless. From here we convert all the steel elements into equivalent concrete elements using the modular ratio and we use the process of geometrical decomposition to find the neutral axis (i.e. $x_c = \text{sum}(A * y) / \text{sum}(A)$ where A are the cross sectional areas of each subdivided element and y is the distance from the top of the beam down to the centroid of each individual element).

Finding Distance to the Neutral Axis (x_c)

Sub Divided Element	Area (A)	Distance to Centroid (y)
Concrete Above Neutral Axis	$(x_c * B) - A_s'$	$x_c / 2$
Bottom Steel	$A_s * \alpha_e$	d
Top Steel	$A_s' * \alpha_e$	d'

$$x_c = \text{sum}(A * y) / \text{sum}(A)$$

$$x_c = \frac{\frac{x_c}{2}(x_c B - A_s') + A_s \alpha_e d + A_s' \alpha_e d'}{x_c B - A_s' + A_s \alpha_e + A_s' \alpha_e}$$

multiply out the bottom of the fraction across the left hand side

$$x_c(x_c B - A_s' + A_s \alpha_e + A_s' \alpha_e) = \frac{x_c}{2}(x_c B - A_s') + A_s \alpha_e d + A_s' \alpha_e d'$$

Factor the α_e terms to simplify the expressions on the left side and right hand side and also multiply out the other brackets with the x_c terms

$$x_c^2 B - x_c A_s' + x_c \alpha_e (A_s + A_s') = \frac{x_c^2 B}{2} - \frac{x_c A_s'}{2} + \alpha_e (A_s d + A_s' d')$$

$$0 = \left(\frac{B}{2} - B\right)x_c^2 + \left(A_s' - \frac{A_s'}{2} - \alpha_e (A_s + A_s')\right)x_c + \alpha_e (A_s d + A_s' d')$$

Get all the equation over onto the right hand side

$$0 = \frac{x_c^2 B}{2} - \frac{x_c A'_s}{2} + \alpha_e (A_s d + A'_s d') - x_c^2 B + x_c A'_s - x_c \alpha_e (A_s + A'_s)$$

Simplify

$$0 = -\frac{B}{2} x_c^2 + \left(\frac{A'_s}{2} - \alpha_e (A_s + A'_s)\right) x_c + \alpha_e (A_s d + A'_s d')$$

We now have a quadratic equation. Use the quadratic formula to solve it

For any quadratic

$$0 = ax^2 + bx + c$$

The solution is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our case the coefficients are as follows:

$$a = -\frac{B}{2} \quad b = \left(\frac{A'_s}{2} - \alpha_e (A_s + A'_s)\right) \quad c = \alpha_e (A_s d + A'_s d')$$

Finding Second Moment of Area

The second moment of area is found by applying the parallel axis theorem which is as follows:

$$I_{\text{total}} = \sum(I) + \sum(A * y_{\text{bar}}^2)$$

where:

- I is the second moment of area for each individual element about it's centroid (for a rectangle it would be $bd^3/12$)

- A is the cross sectional area of each subdivided element

- y_{bar} is the distance from the neutral axis to the centroid of each subdivided element

Sub Divided Element	I	A	y_{bar}	$A * y_{\text{bar}}^2$
Concrete Above Neutral Axis	$1/12 * B * x_c^3$	$(x_c * B) - A'_s$	$x_c / 2$	$\frac{x_c^3 B}{4} - \frac{x_c^2 A'_s}{4}$
Bottom Steel	negligible = 0	$\alpha_e * A_s$	$x_c - d$	$\alpha_e A_s (x_c - d)^2$
Top Steel	negligible = 0	$\alpha_e * A'_s$	$x_c - d'$	$\alpha_e A'_s (x_c - d')^2$

$$I_c = \frac{B x_c^3}{12} + \frac{x_c^3 B}{4} - \frac{x_c^2 A'_s}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A'_s (x_c - d')^2$$

Simplify

$$I_c = \frac{B x_c^3}{3} - \frac{x_c^2 A'_s}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A'_s (x_c - d')^2$$

	Project	N/A	Concrete Beam Design BS EN 1992-1-1		
	Client	N/A	Made by	Date	Job No
	Description	Worked Example	AL	1-11-21	N/A
			Checked	Revision	
	Concrete Beam Design v1.0		N/A	1	

1.0 - ANALYSIS OPTIONS (BOTTOM AND TOP OF BEAM)		
Limit State	Beam Bottom	Beam Top
L/d Deflection Check	INCLUDE	INCLUDE
Crack Width Check	INCLUDE	INCLUDE
1.1 - ANALYSIS OPTIONS (SIDES OF BEAM)		
Limit State	Side Left	Side Right
Bending, Shear & Torsion Checks	INCLUDE	INCLUDE
L/d Deflection Check	INCLUDE	INCLUDE
Crack Width Check	INCLUDE	INCLUDE

2.0 - CONCRETE PROPERTIES @ 28 DAYS (BS EN1992-1-1 Table 3.1)		
Concrete Grade	MANUAL	
Aggregate Type	Limestone	
Maximum Aggregate Size (mm) a _g	20	
Characteristic Cylinder Strength (N/mm ²) f _{ck}	27	
Mean Compressive Strength (N/mm ²) f _{cm}	35	
Mean Tensile Strength (N/mm ²) f _{ctm}	2.70	
Mean Modulus of Elasticity (N/mm ²) E _{cm}	28833	

3.0 - REINFORCING STEEL PROPERTIES		
Characteristic Tensile Strength (N/mm ²) f _{yk}	500	
Modulus of Elasticity (N/mm ²) E _s	205000	

4.0 - CONCRETE BEAM GEOMETRY		
Beam Depth (mm) h	450	
Beam Width (mm) B	600	
Beam Span (mm) L	3500	
Beam Support Conditions	Simply Supported	

5.0 - CONCRETE COVER		
Concrete Cover to Bottom of Beam (mm) c _{bot}	75	
Concrete Cover to Top of Beam (mm) c _{top}	40	
Concrete Cover to Side Left of Beam (mm) c _{sideL}	50	
Concrete Cover to Side Right of Beam (mm) c _{sideR}	50	

6.0 - CRACK WIDTH INPUTS (IF REQUIRED)		
Allowable Crack Width (mm) w _k	0.3	
Type of Rebar (for calculation of k ₁ factor)	High Bond Bars	
Type of Loading (for calculation of k ₂ factor)	Bending	
Number of Days when Considering Short Term Loading t	20	
s factor for cement type	0.2	
Creep Factor for long term loading (t > 28) days φ _c	1.857142	

7.0 - INTERNAL FORCES		
	ULS	SLS
Bottom Moments M _{Ed,bot} (kNm)	50.0	35.0
Top Moments M _{Ed,top} (kNm)	25.0	15.0
Side Left Moment M _{Ed,sideL} (kNm)	10.0	5.0
Side Right Moment M _{Ed,sideR} (kNm)	10.0	5.0
Major Axis Shear V _{y,Ed} (kN)	110.0	---
Minor Axis Shear V _{z,Ed} (kN)	55.0	---
Torsion T _{Ed} (kNm)	7.0	---

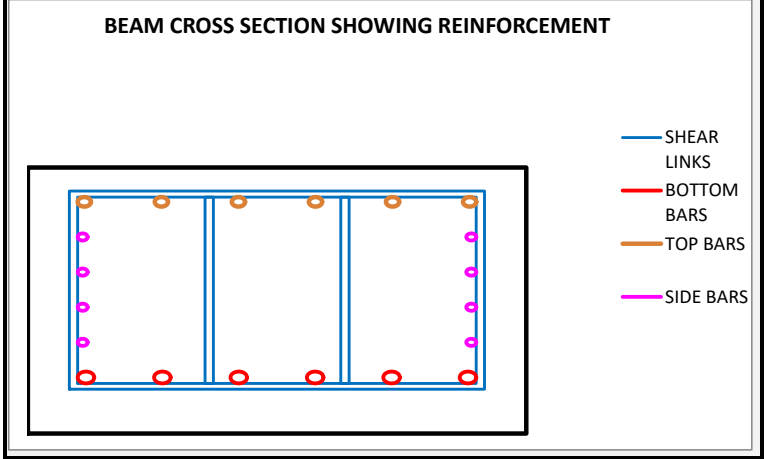
11.0 - CHECK MOMENT CAPACITY OF BEAM AT ULS (AND MIN / MAX AREAS OF STEEL - BS EN1992-1-1 SECTION 6.1)						
Variables	Bottom	Top	SideL	SideR	Formula	
K Factor	0.0245	0.0100	0.0029	0.0029	K = M / (f _{ck} * B * d ²) (M: applied ULS Moment)	
Lever Arm (mm) Z	337.3	372.4	507.3	507.3	Z = min(0.95d ; d * [0.5 + √{0.25 - 3K/3.4}])	
Minimum Area of Reinforcement (mm ² /m') A _{s,min}	299.1	330.2	337.4	337.4	A _{s,min} = max(0.26f _{ctm} /f _{yk} ; 0.0013) * B * d	
Area of Reinforcement Required (mm ² /m') A _{s,req}	340.8	330.2	337.4	337.4	A _{s,req} = max(M / [0.87 * f _{yk} * z]) ; A _{s,min}	
Area of Reinforcement Provided (mm ² /m') A _{s,prov}	1885.0	1206.4	452.4	452.4	See Section 8.0 above on this calculation spreadsheet	
Utilisation %	18%	27%	75%	75%	A _{s,req} / A _{s,prov}	
Maxmium Area of Reinforcement (mm ² /m') A _{s,max}	10800	10800	10800	10800	A _{s,max} = 0.04 * B * h	
Utilisation %	17%	11%	4%	4%	A _{s,prov} / A _{s,max}	

8.0 - REINFORCING STEEL (MAIN BARS)					
	Diametre	Number	Spacing	Clear Dist	Area
	φ	n	S	S'	A _{s,prov}
	mm	---	mm	mm	mm ²
Bottom	20	6	92.00	72.00	1884.96
Top	16	6	92.80	76.80	1206.37
Side Left	12	4	59.40	43.40	452.39
Side Right	12	4	59.40	43.40	452.39
Rebar Mass (not including laps, chairs, spacers or Ubars):					
Bottom Steel: 14.8kg/m			Side Left: 3.55kg/m		
Top Steel: 9.47kg/m			Side Right: 3.55kg/m		

8.1 - REINFORCING STEEL (LINKS)					
	Diametre	Spacing	Legs	Number	Area
	φ	S	n _{legs}	of Links	A _{s,prov}
	mm	mm	---	---	mm ²
Links	10	200	4	18	314.16

Rebar Mass (not including laps, chairs, spacers or Ubars):					
Main Square Links: 6.84kg/m			Leg Links: 4.44kg/m		

8.2 - DEPTHS TO REINFORCEMENT	
Depth to Bottom Steel (mm) d _{bot}	355.00
Depth to Top Steel (mm) d _{top}	392.00
Depth to Side Left Steel (mm) d _{sideL}	534.00
Depth to Side Right Steel (mm) d _{sideR}	534.00
Total Rebar Mass: 42.65kg/m	
Rebar Density within Beam: 157.98kg/m3	



9.0 - GEOMETRIC CHECKS (REBAR SPACING) BS EN1992-1-1 8.2(2)	
Bottom Bars	Clear Spacing OK: 72mm <= 25mm
Top Bars	Clear Spacing OK: 77mm <= 25mm
Side Left Bars	Clear Spacing OK: 43mm <= 25mm
Side Right Bars	Clear Spacing OK: 43mm <= 25mm
Verdict	Bar Spacing OK

10.0 - CHECK MAXIMUM MOMENT CAPACITY OF BEAM		
Bottom (kNm)	340.95	OK
Top (kNm)	415.72	OK
Side Left (kNm)	771.46	OK
Side Right (kNm)	771.46	OK
$M_{max} = 0.167 * f_{ck} * B * d^2$		

12.0 - CHECK DEFLECTION (L/d CHECK - BS EN1992-1-1 SECTION 7.4.2)

Variables	Bottom	Top	SideL	SideR	Formula
K Factor to account for different support systems	1.0	1.0	1.0	1.0	BS EN1992-1-1: Table 7.4N
Reference Reinforcement Ratio ρ_0	0.00520	0.00520	0.00520	0.00520	$\rho_0 = 10^{-3} \sqrt{f_{ck}}$
Required Tension Reinforcement Ratio ρ	0.00160	0.00140	0.00140	0.00140	$\rho = A_{s,req} / (B * d)$
Required Compression Reinforcement Ratio ρ'	0	0	0	0	Zero because we're not using compression rebar
L/d basic (BS EN1992-1-1 Section 7.4.2 Equations 7.16a & 7.16b)	92.3	113.7	113.7	113.7	$\frac{L}{d} = K[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}(\frac{\rho_0}{\rho} - 1)^{\frac{3}{2}}] \quad \rho \leq \rho_0$ $\frac{L}{d} = K[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}] \quad \rho > \rho_0$
Stress in the Steel (N/mm ²) σ_s	56.1	84.9	231.2	231.2	$\sigma_s = (310 * f_{yk} * A_{s,req}) / (500 * A_{s,prov})$ Eq - 7.17
Additional Factor for Long Span Beams (L > 8.5m) γ_{LONG}	1.000	1.000	1.000	1.000	$\gamma_{LONG} = 7m / \text{Span in meters (for Span > 7m)}$
Allowable L/d	510.65	415.21	152.40	152.40	L/d allowable = $\gamma_{LONG} * (L/d \text{ basic}) * (310 / \sigma_s)$
Actual L/d	9.86	8.93	6.55	6.55	L/d actual
Utilisation %	2%	2%	4%	4%	(L/d actual) / (L/d allowable)

13.0 - CHECK SHEAR (BS EN1992-1-1 SECTION 6.2.2)

Variables	Bottom	Top	SideL	SideR	Formula
Partial Material Factor for Concrete γ_c	1.5	1.5	1.5	1.5	BS EN1992-1-1 Table 2.1N
$C_{Rd,c}$	0.12	0.12	0.12	0.12	$C_{Rd,c} = 0.18 / \gamma_c$ UK NA to BS EN1992-1-1 6.2.2(1)
k	1.751	1.714	1.612	1.612	$k = 1 + \sqrt{200/d} \leq 2.0$ 6.2.2
ρ_l	0.0088	0.0051	0.0019	0.0019	$\rho_l = A_{s,prov} / (B * d) \leq 0.02$ 6.2.2
v_{min}	0.421	0.408	0.372	0.372	$v_{min} = 0.035k^{3/2} * f_{ck}^{1/2}$ Eq - 6.3N
Shear Strength of Concrete (kN) $V_{Rd,c}$	128.88	116.19	89.44	89.44	$V_{Rd,c} = \max\{[C_{Rd,c}k(100\rho_l f_{ck})^{1/3}Bd ; v_{min}Bd]\}$ Eq 6.2a, 6.2b
Cross Sectional Area of Shear Reinforcement A_{sw}	314.2	314.2	157.1	157.1	$A_{sw} = (\pi\phi^2)/4 * \text{number of legs}$
Strength Reduction Factor for Concrete Cracked in Shear v_1	0.535	0.535	0.535	0.535	$v_1 = v = 0.6[1 - \{f_{ck}/250\}]$ Eq - 6.6N & UK National Annex
Coefficient for State of Stress in Compression Chord α_{cw}	1.0	1.0	1.0	1.0	$\alpha_{cw} = 1$ (no prestress BS EN1992-1-1 6.2.3(3) Note 3)
Lever Arm (mm) Z	337.3	372.4	507.3	507.3	Z found previously or if no moment is added use 0.9d
Design Strength of Concrete in Compression (N/mm ²) f_{cwd}	18.000	18.000	18.000	18.000	$f_{cwd} = \alpha_{cw} * f_{ck} / \gamma_c$ Eq - 3.15 & where $\alpha_{ccw} = 1.0$ UK NA
Angle between concrete strut and beam axis (degrees) θ	21.8	21.8	21.8	21.8	$\theta = \min(\max[\frac{1}{2}\sin^{-1}(\min[\frac{2V_{Ed}}{\alpha_{cw}f_{cwd}V_1}, 1]), 21.8^\circ], 45^\circ)$ $V_{Ed} = V_{Ed} / (B * z)$
Maximum Shear Capacity of Beam (kN) $V_{Rd,max}$	974.7	1076.3	1099.6	1099.6	$V_{Rd,max} = [\alpha_{cw} * B * Z * v_1 * f_{cwd}] / [\cot(45) + \tan(45)]$ Eq - 6.9
Is Shear Design Possible?	YES	YES	YES	YES	Is $V_{Ed} \leq V_{Rd,max}$?
Spacing of the Stirrups (mm) s	200	200	200	200	User input from section 8.1 on this spreadsheet
Partial Material Factor for Shear Link Rebar γ_s	1.15	1.15	1.15	1.15	BS EN1992-1-1 Table 2.1N
Design Yield Strength of Shear Reinforcement (N/mm ²) f_{ywd}	434.783	434.783	434.783	434.783	$f_{ywd} = f_{yk} / \gamma_s$
Capacity of Current Shear Link Arrangement (kN) $V_{Rd,s}$	575.9	635.9	433.1	433.1	$V_{Rd,s} = (A_{sw}/s) * Z * f_{ywd} * \cot(\theta)$
Utilisation for Major or Minor Axis Shear %	19%	17%	13%	13%	$V_{Ed} / V_{Rd,s}$
Ratio of Shear Reinforcement ρ_w	0.002618	0.002618	0.001745	0.001745	$\rho_w = A_{sw} / (s * B * \sin(90^\circ))$ Eq - 9.4
Minimum Ratio of Shear Reinforcement $\rho_{w,min}$	0.000831	0.000831	0.000831	0.000831	$\rho_{w,min} = (0.08\sqrt{f_{ck}})/f_{yk}$ Eq - 9.5N & UK National Annex
Utilisation for Minimum Area of Shear Reinforcement %	32%	32%	48%	48%	$\rho_{w,min} / \rho_w$

14.0 - CHECK CRACK WIDTHS (BS EN1992-1-1 SECTION 7.3)

14.1 - CONSTANTS

Variables	Bottom	Top	SideL	SideR	Formula
Factor for bond between bars and concrete - k_1	0.8	0.8	0.8	0.8	0.8 High Bond, 1.6 Plain Bars BS EN1992-1-1 7.3.4
Factor for distribution of strain - k_2	0.5	0.5	0.5	0.5	0.5 Bending, 1.0 Pure Tension BS EN1992-1-1 7.3.4
k_3	3.4	3.4	3.4	3.4	Recommended Value: NA to BS EN1992-1-1
k_4	0.425	0.425	0.425	0.425	Recommended Value: NA to BS EN1992-1-1
Depth to Reinforcement in Tension (mm) d	355.00	392.00	534.00	534.00	from user inputs
Depth to Reinforcement in Compression (mm) d'	58.00	95.00	66.00	66.00	from user inputs
Provided Area of Steel in Tension Zone (mm ² /m') A_s	1885	1206	452	452	from user inputs
Provided Area of Steel in Compression Zone (mm ² /m') A'_s	1206	1885	452	452	from user inputs
Mean Modulus of Elasticity of Concrete (N/mm ²) $E_{cm(t)}$	28517	28517	28517	28517	$E_{cm(t)} = E_{cm} * e^{0.3s[1 - \sqrt{\frac{28}{f_{ck}}}]}$ Combine Eq 3.1, 3.2 & 3.5
Modulus of Elasticity of Concrete with Creep (N/mm ²) E_{eff}	10091	10091	10091	10091	$E_{eff} = (E_{cm}) / (1 + \phi_c)$ BS EN1992-1-1 Eq 7.20
Modular Ratio without creep α_e	7.189	7.189	7.189	7.189	$\alpha_e = E_s / E_{cm(t)}$
Modular Ratio with Creep $\alpha_{e,creep}$	20.314	20.314	20.314	20.314	$\alpha_{e,creep} = E_s / E_{eff}$
Bar Diameter ϕ	20.00	16.00	12.00	12.00	from user inputs

14.2 - UNCRACKED SECTION ASSESSMENT - WITHOUT CREEP (BS EN1992-1-1 SECTION 7.3.4)

Uncracked Neutral Axis Depth (mm) x_u	226.08	223.92	300.00	300.00	$x_u = \frac{\frac{h}{2}(Bh - A_s - A'_s) + \alpha_e A_s d + \alpha_e A'_s d'}{(Bh - A_s - A'_s) + \alpha_e A_s + \alpha_e A'_s}$
Uncracked Second Moment of Area (mm ⁴) I_u	5.0E+09	5.0E+09	8.5E+09	8.5E+09	$I_u = \frac{Bh^3}{12} + (Bh - A_s - A'_s)(x_u - \frac{h}{2})^2 + \alpha_e A_s (x_u - d)^2 + \alpha_e A'_s (x_u - d')^2$
Stress in the Concrete (N/mm ²) σ_c	0.898	0.502	0.138	0.138	$\sigma_c = M_{SL5} * (d - x_u) * (1 / I_u)$ Engineers Bending Eq
Stress in the Steel (N/mm ²) σ_s	6.453	3.606	0.995	0.995	$\sigma_s = \sigma_c * \alpha_e$ Engineers Bending Eq
Effective Area of Concrete in Tension (mm ²) $A_{c,eff}$	42898	44010	44548	44548	$A_{c,eff} = \min(2.5[h-d] ; (h-x_u)/3 ; h/2) * B - A_s$ 7.3.2(3)
Ratio of Steel to Effective Area of Concrete $\rho_{p,eff}$	0.04394	0.02741	0.01016	0.01016	$\rho_{p,eff} = A_s / A_{c,eff}$ Eq - 7.10
Maximum Crack Spacing (mm) $S_{r,max}$	332.38	235.23	370.88	370.88	$S_{r,max} = k_3 c + (k_1 k_2 k_4 \phi_{eq} / \rho_{p,eff})$ c = cover [mm] Eq - 7.11

14.3 - UNCRACKED SECTION ASSESSMENT - WITH CREEP (BS EN1992-1-1 SECTION 7.3.4)

Uncracked Neutral Axis Depth (mm) x_u	227.69	222.31	300.00	300.00	Same formula as in section 14.2 above
Uncracked Second Moment of Area (mm ⁴) I_u	5.9E+09	5.9E+09	9.1E+09	9.1E+09	Same formula as in section 14.2 above
Stress in the Concrete (N/mm ²) σ_c	0.757	0.433	0.128	0.128	$\sigma_c = M_{SLs} * (d - x_u) * (1 / I_u)$ Engineers Bending Eq
Stress in the Steel (N/mm ²) σ_s	15.383	8.787	2.610	2.610	$\sigma_s = \sigma_c * \alpha_{e,creep}$ Engineers Bending Eq
Effective Area of Concrete in Tension (mm ²) $A_{c,eff}$	42578	44331	44548	44548	$A_{c,eff} = \min[2.5[h-d] ; (h-x_u)/3 ; h/2] * B - A_s$ 7.3.2(3)
Ratio of Steel to Effective Area of Concrete $\rho_{p,eff}$	0.04427	0.02721	0.01016	0.01016	$\rho_{p,eff} = A_s / A_{c,eff}$ Eq - 7.10
Maximum Crack Spacing (mm) $S_{r,max}$	331.80	235.95	370.88	370.88	$S_{r,max} = k_3c + (k_1k_2k_4\phi_{eq}/\rho_{p,eff})$ c = cover [mm] Eq - 7.11

14.4 - CRACKED SECTION ASSESSMENT - WITHOUT CREEP (BS EN1992-1-1 SECTION 7.3.4)

Cracked Neutral Axis Depth (mm) x_c	101.84	94.46	80.21	80.21	$0 = -\frac{B}{2}x_c^2 + (\frac{A_s}{2} - \alpha_e(A_s + A'_s))x_c + \alpha_e(A_s d + A'_s d')$ solve for x_c
Cracked Second Moment of Area (mm ⁴) I_c	2.71E+09	2.3E+09	2.0E+09	2.0E+09	$I_c = \frac{Bx_c^3}{3} - \frac{x_c^2 A_s}{4} + \alpha_e A_s (x_c - d)^2 + \alpha_e A'_s (x_c - d')^2$
Stress in the Concrete (N/mm ²) σ_c	3.270	1.912	1.151	1.151	$\sigma_c = M_{SLs} * (d - x_c) * (1 / I_c)$ Engineers Bending Eq
Stress in the Steel (N/mm ²) σ_s	23.510	13.746	8.275	8.275	$\sigma_s = \sigma_c * \alpha_e$ Engineers Bending Eq
Effective Area of Concrete in Tension (mm ²) $A_{c,eff}$	67747	69902	73798	73798	$A_{c,eff} = \min[2.5[h-d] ; (h-x_c)/3 ; h/2] * B - A_s$ 7.3.2(3)
Ratio of Steel to Effective Area of Concrete $\rho_{p,eff}$	0.02782	0.01726	0.00613	0.00613	$\rho_{p,eff} = A_s / A_{c,eff}$ Eq 7.10
Maximum Crack Spacing (mm) $S_{r,max}$	377.20	293.61	502.78	502.78	$S_{r,max} = k_3c + (k_1k_2k_4\phi_{eq}/\rho_{p,eff})$ c = cover [mm] Eq - 7.11

14.5 - CRACKED SECTION ASSESSMENT - WITH CREEP (BS EN1992-1-1 SECTION 7.3.4)

Cracked Neutral Axis Depth (mm) x_c	142.91	130.95	121.32	121.32	Same formula as in section 14.4 above
Cracked Second Moment of Area (mm ⁴) I_c	2.5E+09	2.2E+09	1.9E+09	1.9E+09	Same formula as in section 14.4 above
Stress in the Concrete (N/mm ²) σ_c	2.997	1.812	1.110	1.110	$\sigma_c = M_{SLs} * (d - x_c) * (1 / I_c)$ Engineers Bending Eq
Stress in the Steel (N/mm ²) σ_s	60.887	36.817	22.543	22.543	$\sigma_s = \sigma_c * \alpha_{e,creep}$ Engineers Bending Eq
Effective Area of Concrete in Tension (mm ²) $A_{c,eff}$	59534	62604	71350	71350	$A_{c,eff} = \min[2.5[h-d] ; (h-x_c)/3 ; h/2] * B - A_s$ 7.3.2(3)
Ratio of Steel to Effective Area of Concrete $\rho_{p,eff}$	0.03166	0.01927	0.00634	0.00634	$\rho_{p,eff} = A_s / A_{c,eff}$ Eq 7.10
Maximum Crack Spacing (mm) $S_{r,max}$	362.38	277.15	491.74	491.74	$S_{r,max} = k_3c + (k_1k_2k_4\phi_{eq}/\rho_{p,eff})$ c = cover [mm] Eq - 7.11

14.6 - SHORT TERM LOAD (I.E. NO CREEP) - CRACK WIDTH ASSESSMENT (BS EN1992-1-1 SECTION 7.3.4)

Variables	Bottom	Top	SideL	SideR	Formula
Mean Tensile Strength of the Concrete at t days (N/mm ²) $f_{ct,eff}$	2.603	2.603	2.603	2.603	$f_{ct,eff} = f_{ctm(t)} = f_{ctm} * e^{\alpha_s[1-\sqrt{\frac{28}{t}}]}$ Combine Eq 3.2 & 3.4 $\alpha = 1$
Cracking Moment (kNm) M_c	101.49	77.84	94.06	94.06	$M_c = (f_{ct,eff} * I_u) / (d - x_u)$ engineers bending equation
Is the section cracked?	NO	NO	NO	NO	Is $\sigma_c \geq f_{ct,eff}$ where σ_c is from the uncracked section
Maximum Crack Spacing for (mm) $S_{r,max}$	332.38	235.23	370.88	370.88	from previous assessment
Factor for short duration of load - k_t	0.6	0.6	0.6	0.6	BS EN1992-1-1 Section 7.3.4
Mean Strain in Rebar - Mean Strain in Concrete $\epsilon_{sm} - \epsilon_{cm}$	1.89E-05	1.06E-05	2.91E-06	2.91E-06	$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$ Eq - 7.9
Crack Width (mm) w_k	0.01	0.00	0.00	0.00	$w_k = S_{r,max} * (\epsilon_{sm} - \epsilon_{cm})$ Eq - 7.8
Utilisation %	2%	1%	0%	0%	$w_k / w_{k,allowable}$

14.7 - LONG TERM LOAD (I.E. INCLUDING CREEP) - CRACK WIDTH ASSESSMENT (BS EN1992-1-1 SECTION 7.3.4)

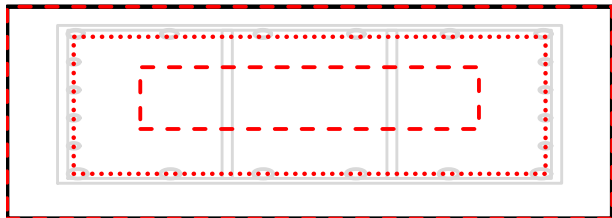
Mean Tensile Strength of the Concrete at t days (N/mm ²) $f_{ct,eff}$	2.70	2.70	2.70	2.70	$f_{ct,eff} = f_{ctm}$ at 28 days
Cracking Moment (kNm) M_c	124.79	93.63	105.07	105.07	$M_c = (f_{ct,eff} * I_u) / (d - x_u)$ engineers bending equation
Is the section cracked?	NO	NO	NO	NO	Is $\sigma_c \geq f_{ctm}$ where σ_c is from the uncracked section
Maximum Crack Spacing for (mm) $S_{r,max}$	331.80	235.95	370.88	370.88	from previous assessment
Factor for short duration of load - k_t	0.4	0.4	0.4	0.4	BS EN1992-1-1 Section 7.3.4
Mean Strain in Rebar - Mean Strain in Concrete $\epsilon_{sm} - \epsilon_{cm}$	4.50E-05	2.57E-05	7.64E-06	7.64E-06	Same formula as in section 14.6 above
Crack Width (mm) w_k	0.01	0.01	0.00	0.00	$w_k = S_{r,max} * (\epsilon_{sm} - \epsilon_{cm})$ Eq - 7.8
Utilisation %	5%	2%	1%	1%	$w_k / w_{k,allowable}$

14.8 - SHORT TERM LOAD (I.E. NO CREEP) - MINIMUM AREA OF TENSILE REINFORCEMENT TO CONTROL CRACKING (BS EN1992-1-1 SECTION 7.3.3)

Area of Concrete within Tension Zone (mm ²) A_{ct}	134350	135650	135000	135000	$A_{ct} = (h - x_u) * B$ (area in tension just before cracking)
Coefficient for Self Equilibrating Stresses k	0.895	0.895	0.790	0.790	$h \leq 300mm \rightarrow k = 1$ (interpolate for interim values) $h > 800mm \rightarrow k = 0.65$ Section 7.3.2
Coefficient for Stress Distribution k_c	0.40	0.40	0.40	0.40	$k_c = 0.4$ Eq 7.2 (simplified - no axial force)
Minimum Area of Steel for Crack Control (mm ²) $A_{s,min}$	250	253	222	222	$A_{s,min} = k_c * k * f_{ct,eff} * A_{ct} * (1/f_{yk})$ Eq - 7.1
Utilisation %	13%	21%	49%	49%	$A_{s,min} / A_s$

14.9 - LONG TERM LOAD (I.E. INCLUDING CREEP) - MINIMUM AREA OF TENSILE REINFORCEMENT TO CONTROL CRACKING (BS EN1992-1-1 SECTION 7.3.3)

Area of Concrete within Tension Zone (mm ²) A_{ct}	133389	136611	135000	135000	$A_{ct} = (h - x_u) * B$ (area in tension just before cracking)
Coefficient for Self Equilibrating Stresses k	0.895	0.895	0.790	0.790	$h \leq 300mm \rightarrow k = 1$ (interpolate for interim values) $h > 800mm \rightarrow k = 0.65$ Section 7.3.2
Coefficient for Stress Distribution k_c	0.40	0.40	0.40	0.40	$k_c = 0.4$ Eq 7.2 (simplified - no axial force)
Minimum Area of Steel for Crack Control (mm ²) $A_{s,min}$	258	264	230	230	$A_{s,min} = k_c * k * f_{ct,eff} * A_{ct} * (1/f_{yk})$ Eq - 7.1
Utilisation %	14%	22%	51%	51%	$A_{s,min} / A_s$

15.0 - CHECK TORSION (BS EN1992-1-1 SECTION 6.3)									
Variables	Bottom	Top	SideL	SideR	Formula				
Applied Design Total Torsion Moment (kNm) T _{Ed}	7	7	7	7	User input from section 7.0 on this spreadsheet				
Depth to reinforcement (mm) d	355.00	392.00	534.00	534.00	Calculated previously in section 8.2 on this spreadsheet				
Cross Sectional Area of Beam (mm ²) A	270000	270000	270000	270000	A = h * B				
Perimetre of Beam (mm) u	2100	2100	2100	2100	u = h + h + B + B				
Distance from Edge of Beam to Centreline of Longitudinal Reinforcement (mm) d'	95.00	58.00	66.00	66.00	d' = cover + link diameter + bar diameter/2				
Effective Wall Thickness (mm) t _{ef}	190.00	128.57	132.00	132.00	t _{ef} = max{ A/u , 2d' } [limited to B/2 or h/2] Section 6.3.2				
Cross Sectional Area Enclosed by the Centrelines of the Connecting Walls (mm ²) A _k	136054.29	136054	136054	136054	A _k = (H - t _{ef,bottom} /2 - t _{ef,top} /2) * (B - t _{ef,sideL} /2 - t _{ef,sideR} /2) Section 6.3.2				
Perimetre of Area Enclosed by the Centrelines of the Connecting Walls (mm) U _k	1517	1517	1517	1517	U _k = 2*(H - t _{ef,bottom} /2 - t _{ef,top} /2) + 2*(B - t _{ef,sideL} /2 - t _{ef,sideR} /2) Section 6.3.2				
Applied Torsional Shear Stress in the Wall (N/mm ²) τ _{t,Ed}	0.135	0.200	0.195	0.195	τ _{t,Ed} = T _{Ed} / (2 * A _k * t _{ef}) Eq - 6.26				
Design Shear Stress from Major Axis Shear (N/mm ²) v _{t,y,Ed}	0.516	0.468	---	---	v _{t,y,Ed} = V _{y,Ed} / (B * d)				
Design Shear Stress from Minor Axis Shear (N/mm ²) v _{t,z,Ed}	---	---	0.229	0.229	v _{t,z,Ed} = V _{z,Ed} / (h * d)				
Strength Reduction Factor for Concrete Cracked in Shear v ₁	0.535	0.535	0.535	0.535	v ₁ = v = 0.6[1-{f _{ck} /250}] Eq - 6.6N & UK National Annex				
Coefficient for State of Stress in Compression Chord α _{cw}	1.0	1.0	1.0	1.0	α _{cw} = 1 (no prestress BS EN1992-1-1 6.2.3 Note 3)				
Partial Material Factor for Concrete γ _c	1.5	1.5	1.5	1.5	BS EN1992-1-1 Table 2.1N				
Design Strength of Concrete in Compression (N/mm ²) f _{cd}	15.30	15.30	15.30	15.30	f _{cd} = α _{cc} * f _{ck} / γ _c Eq - 3.15 & where α _{cc} = 0.85 UK NA				
Design Strength of Concrete in Compression (N/mm ²) f _{cwd}	18.00	18.00	18.00	18.00	f _{cwd} = α _{ccw} * f _{ck} / γ _c Eq - 3.15 & where α _{ccw} = 1.0 UK NA				
Angle between concrete strut and beam axis (degrees) θ _t	21.800	21.800	21.800	21.800	θ = min(max[$\frac{1}{2} \sin^{-1}(\min[\frac{2v_{t,Ed}}{(0.9 + \tau_{t,Ed})\alpha_{cw}f_{c,wd}v_1}, 1])$, 21.8°], 45°)				
Maximum Torsional Capacity of Beam (kNm) T _{Rd,max}	145.98	98.78	101.42	101.42	T _{Rd,max} = 2 * v ₁ * α _{cw} * f _{cd} * A _k * t _{ef} * sin(θ _t)*cos(θ _t) Eq - 6.30				
Maximum Design Shear Capacity (Major & Minor) (kN) V _{Rdt,max}	636.8	703.2	718.4	718.4	V _{Rdt,max} = (α _{cw} * B * 0.9d * v ₁ * f _{cwd}) / (cot(θ _t) + tan(θ _t)) Eq - 6.9				
Interaction Formula - Utilisation %	32%				(T _{Ed} / T _{Rd,max}) + (V _{y,Ed} / V _{Rdt,max}) + (V _{z,Ed} / V _{Rdt,max}) where V _{Rdt,max} is the minimum value Eq - 6.29				
15.1 - CHECK TORSIONAL RESISTANCE OF THE CONCRETE ON IT'S OWN (BS EN1992-1-1 SECTION 6.3)									
Characteristic Axial Strength of Concrete (5% Fractile) (N/mm ²) f _{ctk,0.05}	1.89	1.89	1.89	1.89	f _{ctk,0.05} = 0.7 * f _{ctm} Table 3.1				
Design Axial Strength of Concrete (N/mm ²) f _{ctd}	1.26	1.26	1.26	1.26	f _{ctd} = α _{ct} * f _{ctk,0.05} / γ _c where α _{ct} = 1 UK NA, Eq - 3.16				
Maximum Torsional Capacity of the Beam with No Shear Reinforcement (kNm) T _{Rd,c}	65.143	44.082	45.257	45.257	T _{Rd,c} = 2 * A _k * f _{ctd} * t _{ef} Section 6.3.2(5) & Eq - 6.28				
Shear Strength of Concrete (kN) V _{Rd,c}	128.88	116.19	89.44	89.44	From section 13.0 on this spreadsheet				
Interaction Formula - Utilisation %	172%				(T _{Ed} / T _{Rd,c}) + (V _{y,Ed} / V _{Rd,c}) + (V _{z,Ed} / V _{Rd,c}) where V _{Rd,c} is the minimum value Eq - 6.29				
	WARNING: Additional Reinforcement Required								
Partial Material Factor for Reinforcement γ _s	1.15	1.15	1.15	1.15	BS EN1992-1-1 Table 2.1N				
Design Strength of Reinforcing Steel (N/mm ²) f _{yd}	434.78	434.78	434.78	434.78	f _{yd} = f _{yk} / γ _s				
Required Area of ADDITIONAL Longitudinal Reinforcement (mm ²) A _{s,add,req}	224.47	224.47	224.47	224.47	A _{s,add,req} = [T _{Ed} * U _k * cot(θ _t)] / [2 * A _k * f _{yd}] Eq - 6.28				
Final Required Area of ADDITIONAL Longitudinal Reinforcement (mm ²) A _{s,add,req}	224mm2 (4H10)				Maximum value from row above taken. Additional bars to be placed on the 4 corners of the cross section.				
Required ADDITIONAL Shear Reinforcement for Torsion (this is the area of 1 link leg) (mm ² /m') A _{s,add,link}	23.67	23.67	23.67	23.67	A _{s,add,link} = T _{Ed} / [2 * A _k * f _{yd} * cot(θ _t)]				
Final Required ADDITIONAL Shear Reinforcement for Torsion (this is the area of 1 link leg) (mm ² /m') A _{s,add,link}	23.67				Maximum value from row above taken. Additional Links Should be closed links if possible.				
Maximum Spacing of Additional Torsion Links (mm) s _{w,max}	190	190	190	190	s _{w,max} = min(U _k / 8 , B , h)				
Possible Link Size of ADDITIONAL Torsion Links	H5@175mm c/c (closed links)								
<div><div><div>WALL THICKNESS FOR TORSION DESIGN</div><div><div><div>--- WALL THICKNESS FOR TORSION</div><div>..... CENTRELINE OF WALLS FOR TORSION</div></div></div></div></div>									
16.0 - DESIGN SUMMARY									
Limit State	Bottom	Top	Side Left	Side Right	Limit State	Bottom	Top	Side Left	Side Right
Geometry (Bar Spacings)	OK	OK	OK	OK	Cracking (Short Term)	13%	21%	49%	49%
Bending & Max Reinforcement	18%	27%	75%	75%	Cracking (Long Term)	14%	22%	51%	51%
Deflection (L/d check)	2%	2%	4%	4%	Torsion	WARNING: MORE REBAR REQUIRED			
Shear	32%	32%	48%	48%	DESIGN STATUS	OK			