

Concrete Type

Concrete Grade: C28/35

Aggregate Type: Limestone

Cement Type: Class N

Concrete Properties @ 28 daysCharacteristic Compressive Cylinder Strength $f_{ck} = 28\text{N/mm}^2$ Mean Compressive Strength $f_{cm} = f_{ck} + 8\text{N/mm}^2 = 28\text{N/mm}^2 + 8\text{N/mm}^2 = 36\text{N/mm}^2$ Mean Tensile Strength f_{ctm} if concrete grade \leq C50/60: $f_{ctm} = 0.3f_{ck}^{2/3}$ if concrete grade \geq C50/60: $f_{ctm} = 2.12\ln(1+0.1f_{cm})$ in this instance concrete grade is \leq C50/60 therefore:

$$f_{ctm} = 0.3f_{ck}^{2/3} = 0.3 \cdot (28\text{N/mm}^2)^{2/3} = 2.766\text{N/mm}^2$$

Mean Modulus of Elasticity $E_{cm} = 22(0.1f_{cm})^{0.3} = 22(0.1 \cdot 36\text{N/mm}^2)^{0.3} = 32.3\text{GPa}$

$$E_{cm} = 32308\text{N/mm}^2$$

However the Mean Modulus of Elasticity shown above doesn't include the reduction for the type of aggregate. In this instance we're using limestone and as such a reduction of 10% as per BS EN1992-1-1 Section 3.1.3(2).

$$E_{cm} = 0.9 \cdot 32308\text{N/mm}^2 = 29077\text{N/mm}^2$$

BS EN1992-1-1 Table 3.1

Concrete Properties @ t daysNumber of days in which to assess the concrete properties $t = 20$ dayss factor for the cement type of the concrete $s = 0.25$ (class N)

Class R = 0.2

Class N = 0.25

Class S = 0.38

BS EN1992-1-1 3.1.2(6)

Reduction Factor for Concrete Strength: $\beta_{cc}(t) = \exp[s(1 - \sqrt{\frac{28}{t}})]$

BS EN1992-1-1 Eq 3.2

$$\beta_{cc}(t) = \exp[0.25(1 - \sqrt{\frac{28}{20}})] = 0.955$$

Mean Compressive Strength $f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm} = 0.955 \cdot 36\text{N/mm}^2 = 34.4\text{N/mm}^2$

BS EN1992-1-1 Eq 3.1

Characteristic Cylinder Strength $f_{ck}(t) = f_{cm}(t) - 8\text{N/mm}^2 = 34.4\text{N/mm}^2 - 8\text{N/mm}^2 = 26.4\text{N/mm}^2$

BS EN1992-1-1 3.1.2(5)

Mean Tensile Strength $f_{ctm}(t) = \beta_{cc}(t) \cdot f_{ctm} = 0.955 \cdot 2.766\text{N/mm}^2 = 2.642\text{N/mm}^2$ BS EN1992-1-1 Eq 3.4 where $\alpha = 1$ because $t < 28\text{days}$

Concrete Properties @ t days (Continued)

$$\begin{aligned} \text{Mean Modulus of Elasticity } E_{cm}(t) &= (f_{cm}(t) / f_{cm})^{0.3} * E_{cm}(t) \\ E_{cm}(t) &= (34.4\text{N/mm}^2 / 36\text{N/mm}^2)^{0.3} * 29077\text{N/mm}^2 \\ E_{cm}(t) &= 28683\text{N/mm}^2 \end{aligned}$$

BS EN1992-1-1 Eq 3.5

Shrinkage and Creep Inputs

Cross Sectional Area of the Concrete Element

Assume we're designing a concrete ground beam:

width = 600mm

depth = 450mm

Cross Sectional Area $A_c = 600\text{mm} * 450\text{mm} = 1050\text{mm}^2$

Perimeter of the concrete element in contact with the atmosphere for drying. In this instance assume just the top face is in contact.

 $u = 600\text{mm}$ Age of the concrete under consideration $t = 20\text{days}$ (same as before)Age of concrete at beginning of drying shrinkage (i.e. curing time) $t_s = 3\text{ days}$ Age when concrete is first loaded $t_0 = 18\text{days}$ Ambient Relative Humidity $RH = 50\%$ (UK winter)Ambient Temperature $T = 10^\circ\text{C}$ (celsius - UK winter)

Input Parameter Matrix - (i.e. what parameters are used in what calculations)	Drying Shrinkage	Autogenous Shrinkage	Creep
Cross Sectional Area of the Concrete Element A_c	YES	NO	YES
Perimeter of the concrete element in contact with the atmosphere for drying u	YES	NO	YES
Age of the concrete under consideration t	YES	YES	YES
Age of concrete at beginning of drying shrinkage (i.e. curing time) t_s	YES	NO	NO
Age when concrete is first loaded t_0	NO	NO	YES
Ambient Relative Humidity RH	YES	NO	YES
Ambient Temperature T	NO	NO	YES

Drying Shrinkage

Factor for Relative Humidity $\beta_{RH} = 1.55[1 - (\frac{RH}{100\%})^3]$
BS EN1992-1-1 Eq B.12

$$\beta_{RH} = 1.55[1 - (\frac{50\%}{100\%})^3] = 1.356$$

Factor for Type of Cement $\alpha_{sd1} = 4$

Cement Class S --> $\alpha_{sd1} = 3$

Cement Class N --> $\alpha_{sd1} = 4$

Cement Class R --> $\alpha_{sd1} = 6$

BS EN1992-1-1 B.2

Factor for Type of Cement $\alpha_{sd2} = 0.12$

Cement Class S --> $\alpha_{sd2} = 0.13$

Cement Class N --> $\alpha_{sd2} = 0.12$

Cement Class R --> $\alpha_{sd2} = 0.11$

BS EN1992-1-1 B.2

Basic Drying Shrinkage Strain $\epsilon_{cd,0} = 0.85[(220 + 110\alpha_{ds1}) * \exp(-\alpha_{sd2} \frac{f_{cm}}{f_{cmo}})] * 10^{-6} * \beta_{RH}$
where $f_{cmo} = 10\text{N/mm}^2$

BS EN1992-1-1 Eq B.11

$$\epsilon_{cd,0} = 0.85[(220 + 110 * 4) * \exp(-0.12 \frac{36\text{N/mm}^2}{10\text{N/mm}^2})] * 10^{-6} * 1.356 = 0.000494$$

Notional Size of Cross Section $h_0 = 2A_c / u = (2 * 1050\text{mm}^2) / (600\text{mm}) = 3.5$

BS EN1992-1-1 Eq B.10 (notes)

k_h Factor using Table 3.3

Table 3.3

h_0	k_h
100	1.00
200	0.85
300	0.75
≥ 500	0.70

h_0 is less than 100 therefore
 $k_h = 1.00$

(intermediate values can be
found by interpolation i.e. if h_0
= 150 then $k_h = 0.925$)

Drying Shrinkage Factor $\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{h_0^3}}$
BS EN1992-1-1 Eq 3.10

$$\beta_{ds}(t, t_s) = \frac{(20 - 3)}{(20 - 3) + 0.04\sqrt{3.5^3}} = 0.985$$

Drying Shrinkage (Continued)

Drying Shrinkage Strain at Time t $\epsilon_{cd}(t) = \beta_{ds}(t, t_s) * k_h * \epsilon_{cd,0}$
 BS EN1992-1-1 Eq 3.9 $\epsilon_{cd}(t) = 0.985 * 1.0 * 0.000494 = 0.000487$

Drying Shrinkage Strain at Infinite time $\epsilon_{cd}(\infty) = k_h * \epsilon_{cd,0}$
 BS EN1992-1-1 Eq 3.9 $\epsilon_{cd}(\infty) = 1.0 * 0.000494 = 0.000494$

Autogenous Shrinkage

Autogenous Shrinkage Factor $\beta_{as}(t) = 1 - \exp(-0.2t^{0.5})$
 BS EN1992-1-1 Eq 3.13 $\beta_{as}(t) = 1 - \exp(-0.2 * 20^{0.5}) = 0.591$

Autogenous Shrinkage Strain at Infinite Time
 BS EN1992-1-1 Eq 3.12 $\epsilon_{ca}(\infty) = 2.5(f_{ck} - 10) * 10^{-6}$
 $\epsilon_{ca}(\infty) = 2.5(28N/mm^2 - 10) * 10^{-6} = 0.000045$

Autogenous Shrinkage Strain at Time t $\epsilon_{ca}(t) = \epsilon_{ca}(\infty) * \beta_{as}(t)$
 BS EN1992-1-1 Eq 3.11 $\epsilon_{ca}(t) = 0.000045 * 0.591 = 0.0000266$

Creep

Temperature Adjusted Concrete Age $t_{0,T} = t_0 * \exp(-[\frac{4000}{273 + T} - 13.65])$
 BS EN1992-1-1 Eq B.10 $t_{0,T} = 18 * \exp(-[\frac{4000}{273 + 10^{\circ}C} - 13.65]) = 11.091$

Power Depending on Type of Cement $\alpha = 0$

Cement Class S --> $\alpha = -1$

Cement Class N --> $\alpha = 0$

Cement Class R --> $\alpha = 1$

BS EN1992-1-1 Eq B.9 notes

Modified Age of Loading $t_0 = t_{0,T}(\frac{9}{2 + t_{0,T}^{1.2}} + 1)^{\alpha} \geq 0.5$
 BS EN1992-1-1 Eq B.9 $t_0 = 11.091[\frac{9}{2 + 11.091^{1.2}} + 1]^0 \geq 0.5 = 11.091 \leq 0.5 = 11.091$

Factor for Concrete Strength $\alpha_1 = [35 / f_{cm}]^{0.7} = [35 / 36N/mm^2]^{0.7} = 0.980$

Factor for Concrete Strength $\alpha_2 = [35 / f_{cm}]^{0.2} = [35 / 36N/mm^2]^{0.2} = 0.994$

Factor for Concrete Strength $\alpha_3 = [35 / f_{cm}]^{0.5} = [35 / 36N/mm^2]^{0.5} = 0.986$

BS EN1992-1-1 Eq B.8c

Creep (Continued)

Coefficient for Humidity and Notional Size

BS EN1992-1-1 Eq B.8a & B.8b

$$\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250 \leq 1500 \text{ for } f_{cm} \leq 35\text{N/mm}^2$$

$$\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250\alpha_3 \leq 1500\alpha_3 \text{ for } f_{cm} \geq 35\text{N/mm}^2$$

 $f_{cm} = 36\text{N/mm}^2$ therefore use second equation B.9b

$$\beta_H = 1.5[1 + (0.012 * 50)^{18}]3.5 + 250 * 0.986 \leq 1500 * 0.986$$

$$\beta_H = 251.75 \leq 1479 = 251.75$$

Coefficient for the Development of Creep

BS EN1992-1-1 Eq B.7

 $(t_0$ is the non adjusted age of loading)

$$\beta_c(t, t_0) = \left[\frac{(t - t_0)}{(\beta_H + t - t_0)} \right]^{0.3}$$

$$\beta_c(t, t_0) = \left[\frac{(20 - 18)}{(251.75 + 20 - 18)} \right]^{0.3} = 0.234$$

Coefficient for the Age of Concrete when First Loaded

BS EN1992-1-1 Eq B.5

 $(t_0$ is the adjusted age of loading)

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{0.2})}$$

$$\beta(t_0) = \frac{1}{0.1 + 11.091^{0.2}} = 0.582$$

Coefficient for the Effect of Concrete Strength

BS EN1992-1-1 Eq B.4

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = \frac{16.8}{\sqrt{36\text{N/mm}^2}} = 2.8$$

Coefficient for Relative Humidity

BS EN1992-1-1 Eq B.3a & B.3b

$$\phi_{RH} = 1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \text{ for } f_{cm} \leq 35\text{N/mm}^2$$

$$\phi_{RH} = \left[1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \alpha_1 \right] \alpha_2 \text{ for } f_{cm} > 35\text{N/mm}^2$$

 $f_{cm} = 36\text{N/mm}^2$ therefore use second equation B.3b

$$\phi_{RH} = \left[1 + \frac{1 - 50/100}{0.1\sqrt[3]{3.5}} * 0.980 \right] * 0.994 = 4.202$$

Creep (Continued)

Notional Creep Coefficient
BS EN1992-1-1 Eq B.2

$$\phi_0 = \phi_{RH} * \beta(f_{cm}) * \beta(t_0)$$

$$\phi_0 = 4.202 * 2.8 * 0.582 = 6.848$$

Creep Coefficient at time t
BS EN1992-1-1 Eq B.1

$$\phi(t, t_0) = \phi_0 * \beta_c(t, t_0)$$

$$\phi(t, t_0) = 6.848 * 0.234 = 1.602$$

Creep Coefficient at infinite time $\phi(\infty, t_0) = \phi_0 = 6.848$

Non-Linear Creep

If the applied compressive stress applied to the concrete at time t_0 is greater than $0.45f_{ck}(t_0)$ then non-linear creep will occur. This following checks to see whether this is the case or not for a test case of applied compressive stress equal to 15N/mm^2

Applied Compressive Stress at time = $t_0 = 18\text{days}$

$$\sigma_c = 15\text{N/mm}^2$$

Find Characteristic compressive cylinder strength at t_0

$$\beta_{cc}(t_0) = \exp[s(1 - \sqrt{\frac{28}{t_0}})]$$

$$\beta_{cc}(t_0) = \exp[0.25(1 - \sqrt{\frac{28}{18}})] = 0.94$$

$$f_{ck}(t_0) = (f_{cm} * \beta_{cc}(t_0)) - 8\text{N/mm}^2$$

$$f_{ck}(t_0) = (36\text{N/mm}^2 * 0.94) - 8\text{N/mm}^2 = 25.84\text{N/mm}^2$$

BS EN1992-1-1
Eq 3.2
Eq 3.1
Table 3.1

Is $\sigma_c > 0.45 * f_{ck}(t_0)$

$$15\text{N/mm}^2 > 0.45 * 25.84\text{N/mm}^2$$

$$15\text{N/mm}^2 > 11.628\text{N/mm}^2$$

YES therefore non-linear creep does occur

BS EN1992-1-1
Section 3.1.4(4)

k_σ factor for non-linear creep

BS EN1992-1-1 Section 3.1.4(4)

$$k_\sigma = \frac{\sigma_c}{f_{ck}(t_0)} = \frac{15\text{N/mm}^2}{25.84\text{N/mm}^2} = 0.580$$

Non Linear Creep Coefficient at Infinite Time

BS EN1992-1-1 Eq 3.7

$$\phi_{nl}(\infty, t_0) = \phi(\infty, t_0) \exp(1.5(k_\sigma - 0.45))$$

$$\phi_{nl}(\infty, t_0) = 6.848 * \exp(1.5(0.580 - 0.45)) = 8.322$$

Non-Linear Creep (Continued)

Non Linear Creep Coefficient at Time t
BS EN1992-1-1 Eq 3.7

$$\phi_{nl}(t, t_0) = \phi(t, t_0) \exp(1.5(k_\sigma - 0.45))$$

$$\phi_{nl}(t, t_0) = 1.602 * \exp(1.5(0.580 - 0.45)) = 1.947$$

Early Age Thermal Cracking (CIRIA C660)

Aggregate Type = Limestone (from before)

Coefficient of Thermal Expansion $\alpha_c = 0.009 \text{ } \epsilon / ^\circ\text{C}$

CIRIA C660 Table 4.4 (see below)

Thermal Expansion Coefficient (Browne, 1972)

Coarse Aggregate Group	Thermal Expansion Coefficient (Design Value)
	$\epsilon / ^\circ\text{C}$
Chert or Flint	0.012
Quartzite	0.014
Sandstone	0.0125
Marble	0.007
Siliceous Limestone	0.0105
Granite	0.010
Dolerite	0.0095
Basalt	0.010
Limestone	0.009
Glacial Gravel	0.013
Lytag (Course and Fine)	0.007
Lytag Coarse and Natural Aggregate Fines	0.009

Binder Content

Concrete Grade C28/35

Assume we'll use 50% GGBS (ground granulated blast furnace slag).

From the table below our binder content is between 310kg/m³ and 355kg/m³

The higher the binder content the higher the internal curing temperature of the concrete and hence the more thermal cracking that will occur.

To be conservative use the value 355kg/m³**CIRIA C660 - Table 4.2 - Cement Contents for Different Strength Classes**

Strength Class	Binder Content (kg/m ³)										
	Not Specified	CEM I	Up to 20% Fly Ash	30% Fly Ash	40% Fly Ash	50% Fly Ash	Up to 40% GGBS	50% GGBS	60% GGBS	70% GGBS	80% GGBS
C20/25	275	275	295	300	315	330	275	285	300	325	345
C25/30	300	300	320	325	340	360	300	310	330	355	385
C30/37	340	340	360	365	380	400	340	355	375	410	450
C35/45	380	380	405	410	430	450	380	395	430	480	540
C40/50	410	410	440	445	465	485	410	430	470	530	N/A
C45/55	440	440	470	475	500	525	440	465	515	N/A	N/A
C50/60	475	475	505	515	535	N/A	475	505	N/A	N/A	N/A

Values here should not be used for concrete specification purposes. It's only to be used for calculation of thermal cracking

Note: The shaded values are those which may be necessary without the use of admixtures. However, it is common practice to use water-reducing admixtures for the higher strength classes to enable a reduction in cement content. Values greater than 550 kg/m³ have not been included as they would not normally be permitted.

Formwork = plywood

Thickness of Section $h = 450\text{mm}$ (from before)

Binder content = 355kg/m^3

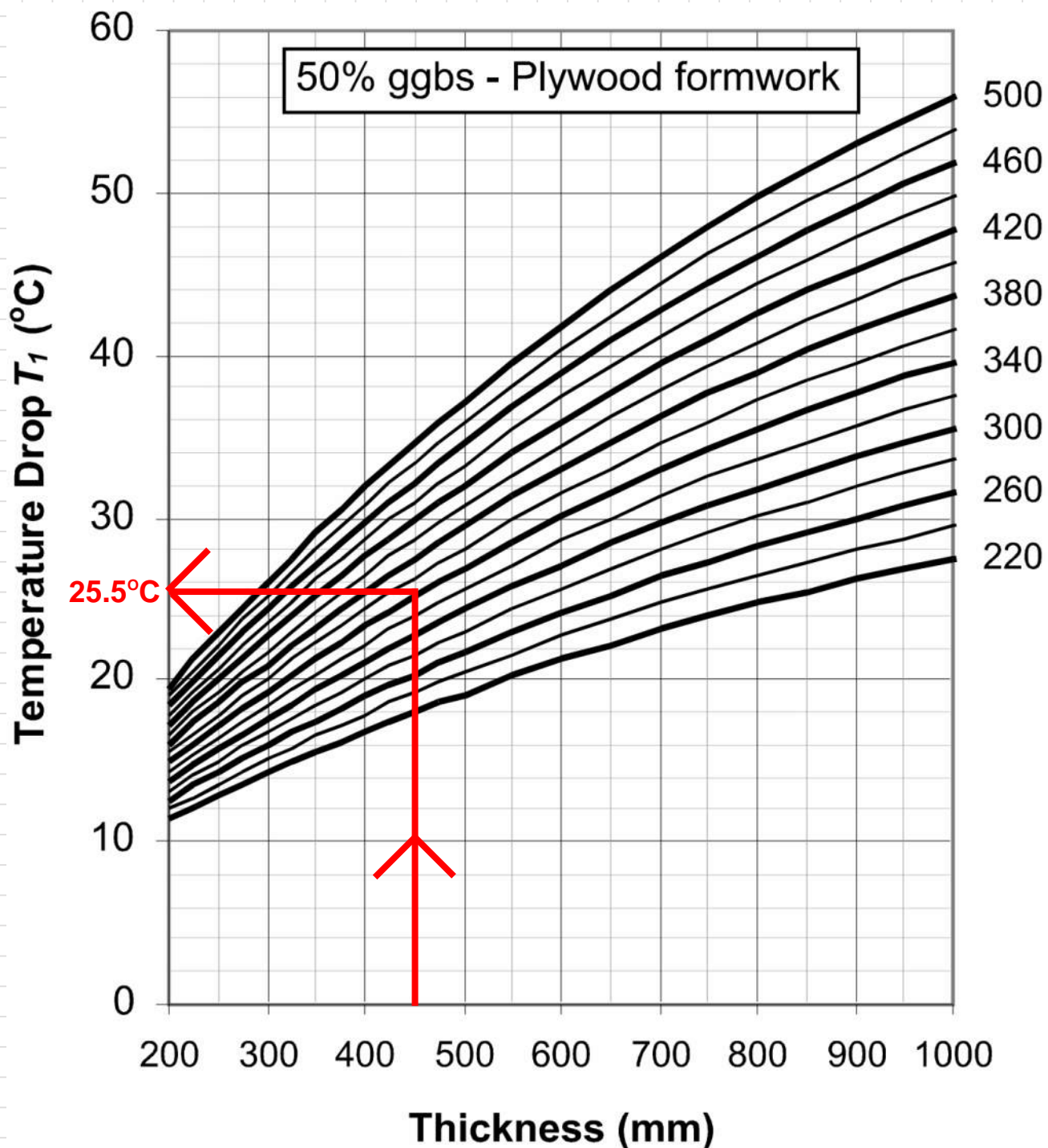


Figure 4.6 from CIRIA C660

From Chart Above Temperature Drop

$T_1 = 25.5^\circ\text{C}$

(This is the difference between the peak temperature of the concrete during hydration (curing) and the ambient surrounding temperature)

Long Term Fall in Temperature

$$T_2 = 10^\circ\text{C}$$

(assuming we're casting in UK winter)

Coefficient for Creep

$$K_1 = 0.65 \text{ (use typical value)}$$

Restraint Factor During Early Thermal Cycle

$$R_1 = 0.5 \text{ (use typical value)}$$

Restraint Factor For Long Term Thermal Movement

$$R_2 = 0.5 \text{ (use typical value)}$$

Restraint Factor For Drying Shrinkage

$$R_3 = 0.5 \text{ (use typical value)}$$

Short Term Thermal Cracking

Autogenous Strain at 3 days

BS EN1992-1-1 Eq 3.11, 3.12, 3.13

$$\epsilon_{ca}(3) = [1 - \exp(-0.2 * \sqrt{3})] * [2.5(28\text{N/mm}^2 - 10)10^{-6}] = 0.0000132$$

Early Age Restrained Strain

CIRIA C660 Section 3.2.1

$$\epsilon_r = K_1 R_1 (\alpha_c T_1 + \epsilon_{ca}(3))$$

$$\epsilon_r = 0.65 * 0.5(0.009 * 25.5 + 0.0000132) = 0.0746$$

Tensile Strength at 3 Days

BS EN1992-1-1 Eq 3.4

$$f_{ctm}(3) = \beta_{cc}(3) * f_{ctm}$$

$$f_{ctm}(3) = \exp(0.25[1 - \sqrt{\frac{28}{3}}]) * 2.766\text{N/mm}^2 = 1.655\text{N/mm}^2$$

Mean Modulus of Elasticity at 3 Days

BS EN1992-1-1 Eq 3.5

$$E_{cm}(3) = \left(\frac{f_{cm}(3)}{f_{cm}}\right)^{0.3} * E_{cm}$$

$$E_{cm}(3) = \left(\frac{21.54\text{N/mm}^2}{36\text{N/mm}^2}\right)^{0.3} * 29077\text{N/mm}^2 = 24925\text{N/mm}^2$$

Tensile Strain Capacity

CIRIA C660 Eq 4.7

$$\epsilon_{ctu} = 1.01 \frac{f_{ctm}(3)}{E_{cm}(3) * 10^{-6}} + 8.4 = 1.01 \frac{1.655\text{N/mm}^2}{24925\text{N/mm}^2 * 10^{-6}} + 8.4 = 75.466\mu\epsilon$$

$$\epsilon_{ctu} = 0.0755$$

Short Term Thermal Cracking (Continued)

Is the restrained strain less than the tensile strain capacity?

$$\epsilon_r < \epsilon_{ctu}$$

0.0746 < 0.0755 --> YES therefore no short term thermal cracking occurs

Long Term Thermal Cracking

Autogenous Strain at 28 days

BS EN1992-1-1 Eq 3.11, 3.12, 3.13

$$\epsilon_{ca}(28) = [1 - \exp(-0.2 * \sqrt{(28)})] * [2.5(28 N/mm^2 - 10)10^{-6}] = 0.0000294$$

Drying Shrinkage at infinite time (calculated previously)

$$\epsilon_{cd}(\infty) = 1.0 * 0.000494 = 0.000494$$

Long Term Restrained Strain

CIRIA C660 Eq 3.2

$$\epsilon_r = K_1([\alpha_c T_1 + \epsilon_{ca}]R_1 + \alpha_c T_2 R_2 + \epsilon_{cd} R_3)$$

$$\epsilon_r = 0.65([0.009 * 25.5 + 0.0000294]0.5 + 0.009 * 10 * 0.5 + 0.000494 * 0.5) = 0.104$$

Tensile Strain Capacity

CIRIA C660 Eq 4.7

(values of f_{ctm} and E_{cm} at 28 days calculated previously)

$$\epsilon_{ctu} = 1.01 \frac{f_{ctm}(28)}{E_{cm}(28) * 10^{-6}} + 8.4 = 1.01 \frac{2.766 N/mm^2}{29077 N/mm^2 * 10^{-6}} + 8.4 = 104.5 \mu\epsilon$$

$$\epsilon_{ctu} = 0.105$$

Is the restrained strain less than the tensile strain capacity?

$$\epsilon_r < \epsilon_{ctu}$$

0.104 < 0.105 --> YES therefore no long term thermal cracking occurs

if thermal cracking did occur you'd use the following equation to find the thermal strain contributing to cracking (CIRIA C660 Eq 3.5)

$$\epsilon_{cr} = \epsilon_r - 0.5\epsilon_{ctu}$$

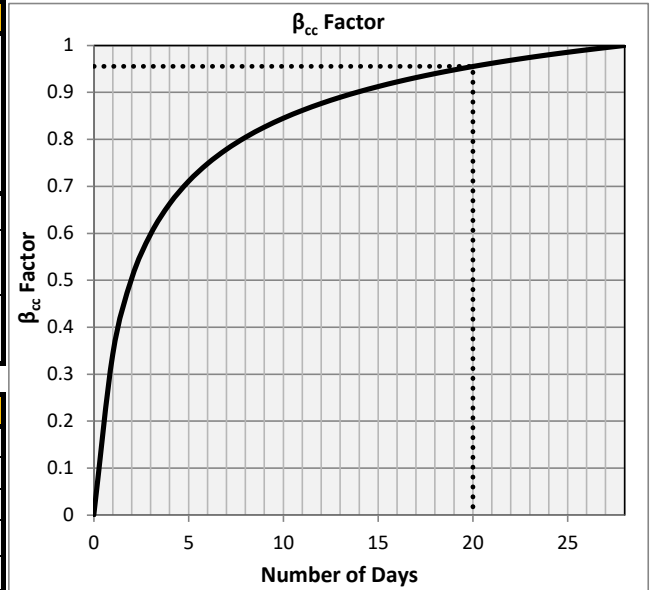
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	Client	N/A	Made by	Date	Job No
	Description	Worked Example 1	AL	6-9-21	N/A
			Checked	Revision	
	Concrete Properties v1.0		N/A	1	

1.0 - CONCRETE PROPERTIES @ 28 DAYS (BS EN1992-1-1 TABLE 3.1)

Concrete Grade	C28/35	
Aggregate Type	Limestone	
Cement Type	Class N	
Characteristic Cylinder Strength (N/mm ²) f_{ck}	28	
Mean Compressive Strength (N/mm ²) f_{cm}	36	$f_{cm} = f_{ck} + 8 \text{ N/mm}^2$
Mean Tensile Strength (N/mm ²) f_{ctm}	2.766	$f_{ctm} = 0.3f_{ck}^{2/3} \leq \text{C50/60}$ $f_{ctm} = 2.12 \ln(1+0.1f_{cm}) > \text{C50/60}$
Mean Modulus of Elasticity (N/mm ²) E_{cm}	29077	$E_{cm} = 22(0.1f_{cm})^{0.3}$ & reduce for aggregate type as needed (3.1.3)

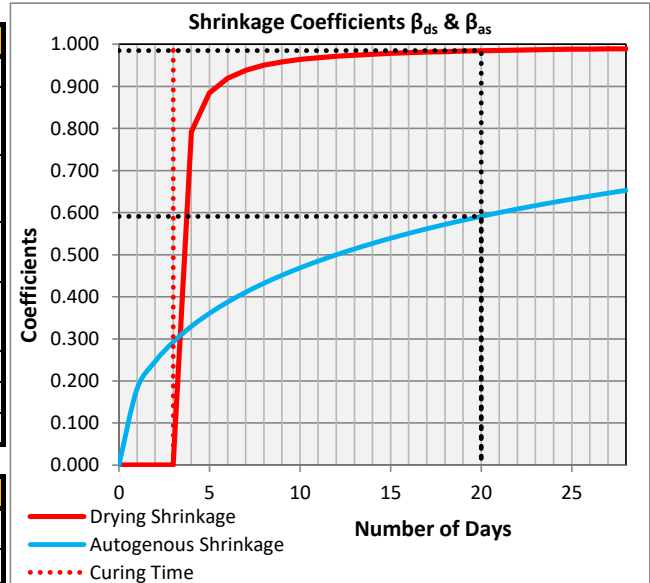
2.0 - CONCRETE PROPERTIES @ t DAYS (t ≤ 28days)

Include Concrete Properties Calculations?	INCLUDE	
Number of Days t	20	Only used for section 2.0
s factor for cement type	0.25	BS EN1992-1-1 3.1.2(6)
$\beta_{cc}(t)$	0.955	$\beta_{cc}(t) = \exp[s(1-(28/t)^{0.5})]$ Eq - 3.2
Mean Compressive Strength (N/mm ²) $f_{cm}(t)$	34.39	$f_{cm}(t) = \beta_{cc}(t) * f_{cm}$ Eq - 3.1
Characteristic Cylinder Strength (N/mm ²) $f_{ck}(t)$	26.39	$f_{ck}(t) = f_{cm}(t) - 8 \text{ N/mm}^2$ 3.1.2(5)
Mean Tensile Strength (N/mm ²) $f_{ctm}(t)$	2.642	$f_{ctm}(t) = \beta_{cc}(t) * f_{ctm}$ Eq - 3.4
Mean Modulus of Elasticity (N/mm ²) $E_{cm}(t)$	28681	$E_{cm}(t) = (f_{cm}(t)/f_{cm})^{0.3} * E_{cm}$ Eq - 3.5



3.0 - SHRINKAGE & CREEP INPUTS

Include Shrinkage & Creep Calculations?	INCLUDE	
Cross Sectional Area of the Concrete Element (mm ²) A_c	1050	for drying shrinkage & creep
Perimetre of the Cross Section Exposed to Drying (mm) u	600	for drying shrinkage & creep
Age of Concrete Under Consideration (days) t	20	for shrinkage & creep calculations
Age of Concrete at beginning of drying shrinkage (normally at end of curing) t_s	3	for drying shrinkage calculations
Age of Concrete when First Loaded t_0	18	for creep calculations
Ambient Relative Humidity % RH	50%	for creep & drying shrinkage
Ambient Temperature (°C) T	10.00	for creep calculations (celsius)



3.1 - DRYING SHRINKAGE

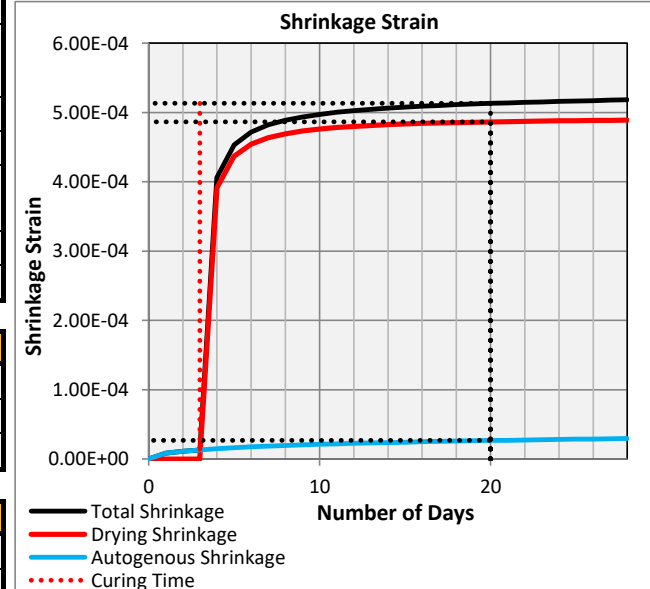
Factor for Relative Humidity β _{RH}	1.356	$\beta_{RH} = 1.55[1-(RH/100\%)^3]$ Eq B.12
Factor for Type of Cement α _{sd1}	4	BS EN1992-1-1 Appendix B
Factor for Type of Cement α _{sd2}	0.12	BS EN1992-1-1 Appendix B
Basic Drying Shrinkage Strain ε _{cd,0}	4.94E-04	$\epsilon_{cd,0} = 0.85[(220+110\alpha_{sd1})\exp(-\alpha_{sd2} \frac{f_{cm}}{f_{cmo}})] * 10^{-6} * \beta_{RH}$ ($f_{cmo} = 10 \text{ MPa}$) Eq - B.11
Notional Size of the Cross Section (mm) h ₀	3.50	$h_0 = 2A_c / u$ Eq - 3.10
k _h Factor	1	Table 3.3
Drying Shrinkage Factor β _{ds} (t,t _s)	0.985	$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{h_0^3}}$ Eq - 3.10
Drying Shrinkage Strain at Time t ε _{cd} (t)	4.86E-04	$\epsilon_{cd}(t) = \beta_{ds}(t, t_s) * k_h * \epsilon_{cd,0}$ Eq - 3.9
Drying Shrinkage Strain, Infinite Time ε _{cd} (∞)	4.94E-04	$\epsilon_{cd}(\infty) = k_h * \epsilon_{cd,0}$ 3.1.4

3.2 - AUTOGENOUS SHRINKAGE

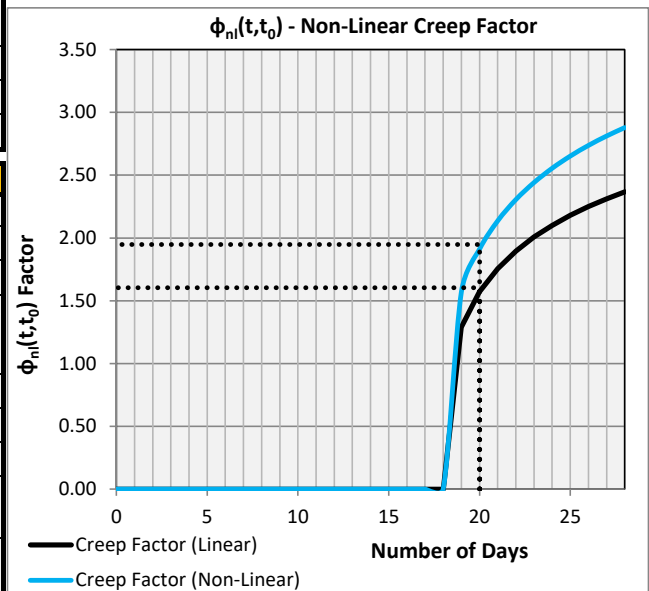
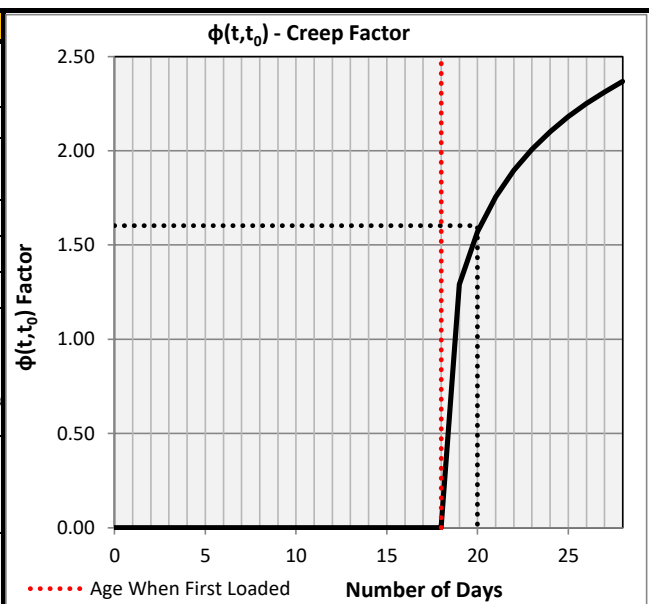
Autogenous Shrinkage Factor β _{as} (t)	0.591	$\beta_{as}(t) = 1 - \exp(-0.2t^{0.5})$ Eq - 3.13
Autogenous Shrinkage Strain at Time t ε _{ca} (t)	2.66E-05	$\epsilon_{ca}(t) = \beta_{as}(t) * \epsilon_{ca}(\infty)$ Eq - 3.11
Autogenous Shrinkage at Infinite Time ε _{ca} (∞)	4.50E-05	$\epsilon_{ca}(\infty) = 2.5(f_{ck} - 10)10^{-6}$ Eq - 3.12

3.3 - TOTAL SHRINKAGE (DRYING + AUTOGENOUS)

Total Shrinkage at time t ε _{cs} (t)	5.13E-04	$\epsilon_{cs}(t) = \epsilon_{cd}(t) + \epsilon_{ca}(t)$ Eq - 3.8
Total Shrinkage at Infinite time ε _{cs} (∞)	5.39E-04	$\epsilon_{cs}(\infty) = \epsilon_{cd}(\infty) + \epsilon_{ca}(\infty)$ Eq - 3.8



4.0 - CREEP		
Temperature Adjusted Concrete Age $t_{0,T}$ (T = ambient temperature, t = time in days)	11.091	$t_{0,T} = t_0 * \exp\{-[(4000/(273+T))-13.65]\}$ Eq - B.10
Power Depending on Cement Type α	0	BS EN1992-1-1 Appendix B
Modified Age of Loading t_0	11.091	$t_0 = t_{0,T}(\frac{9}{2 + t_{0,T}^{1.2}} + 1)^\alpha \geq 0.5$ Eq - B.9
Factor for Concrete Strength α_1	0.980	$\alpha_1 = [35/f_{cm}]^{0.7}$ Eq - B.8c
Factor for Concrete Strength α_2	0.994	$\alpha_2 = [35/f_{cm}]^{0.2}$ Eq - B.8c
Factor for Concrete Strength α_3	0.986	$\alpha_3 = [35/f_{cm}]^{0.5}$ Eq - B.8c
Coefficient for Humidity & Notional Size β_H	251.75	$f_{cm} \leq 35\text{N/mm}^2$: $\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250 \leq 1500$ $f_{cm} > 35\text{N/mm}^2$: $\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250\alpha_3 \leq 1500\alpha_3$ Eq - B.8a & B.8b
Coefficient for Development of Creep $\beta_c(t, t_0)$	0.234	$\beta_c(t, t_0) = [(t - t_0) / (\beta_H + t - t_0)]^{0.3}$ where t_0 is the non modified age of loading (user input) Eq - B.7
Coefficient for Effect of Concrete Age at Loading $\beta(t_0)$	0.582	$\beta(t_0) = 1 / (0.1 + t_0^{0.2})$ where t_0 is the modified age of loading Eq - B.5
Coefficient for Effect of Concrete Strength $\beta(f_{cm})$	2.800	$\beta(f_{cm}) = 16.8 / \sqrt{f_{cm}}$ Eq - B.4
Coefficient for Effect of Relative Humidity ϕ_{RH}	4.205	$f_{cm} \leq 35\text{N/mm}^2$: $\phi_{RH} = 1 + \frac{1 - RH/100}{0.1 \sqrt[3]{h_0}}$ $f_{cm} > 35\text{N/mm}^2$: $\phi_{RH} = [1 + \frac{1 - RH/100}{0.1 \sqrt[3]{h_0}} \alpha_1] \alpha_2$ Eq - B.3a & B.3b
Notional Creep Coefficient ϕ_0	6.853	$\phi_0 = \phi_{RH} * \beta(f_{cm}) * \beta(t_0)$ Eq - B.2
Creep Coefficient at Time t $\phi(t, t_0)$	1.603	$\phi(t, t_0) = \phi_0 * \beta_c(t, t_0)$ Eq - B.1
Creep Coefficient at Infinite Time $\phi(\infty, t_0)$	6.853	$\phi(\infty, t_0) = \phi_0$



4.1 - NON LINEAR CREEP		
Include Checks for Non-Linear Creep	INCLUDE	
Applied Compressive Stress at t_0 : σ_c (N/mm ²)	15	
$\beta_{cc}(t_0)$	0.940	$\beta_{cc}(t_0) = \exp(s[1 - (28/t_0)^{0.5}])$ Eq - 3.2
Characteristic Compressive Cylinder Strength at time t_0 $f_{ck}(t_0)$ (N/mm ²)	25.842	$f_{ck}(t_0) = (\beta_{cc} * f_{cm}) - 8\text{MPa}$ Eq - 3.1 & Table 3.1
$0.45f_{ck}(t_0)$	11.629	BS EN1992-1-1 Section 3.1.4(4)
Is $\sigma_c > 0.45f_{ck}(t_0)$	YES	Account for Creep Non-Linearity
Stress Strength Ratio k_σ	0.580	$k_\sigma = \sigma_c / f_{ck}(t_0)$ 3.1.4(4)
Non Linear Creep Coefficient at Infinite Time $\phi_{nl}(\infty, t_0)$	8.334	$\phi_{nl}(\infty, t_0) = \phi(\infty, t_0) * \exp(1.5(k_\sigma - 0.45))$ Eq 3.7
Non Linear Creep Coefficient at Time t $\phi_{nl}(t, t_0)$	1.949	$\phi_{nl}(t, t_0) = \phi(t, t_0) * \exp(1.5(k_\sigma - 0.45))$ Eq 3.7

5.0 - EARLY AGE THERMAL CRACKING (CIRIA C660 - EARLY-AGE THERMAL CRACK CONTROL)		
Include Early Age Thermal Cracking?	INCLUDE	
Aggregate Type (for Calculation of α_c)	Limestone	Determines the Coefficient of Thermal Expansion
Coefficient of Thermal Expansion ($\epsilon/^\circ\text{C}$) α_c	0.009	CIRIA C660 - Table 4.4 - Originally from Browne 1972
Binder Content (kg/m ³)	355	CIRIA C660 - Table 4.2 (used to find T_1 using Figures 4.5, 4.6, 4.7)
Thickness of Section (mm) h	450	Used with Figures 4.5, 4.6 & 4.7 in CIRIA C660 to find T_1
Type of Formwork	Plywood	Used with Figures 4.5, 4.6 & 4.7 in CIRIA C660 to find T_1
Temperature Drop ($^\circ\text{C}$) T_1	25.5	CIRIA C660 - Figures 4.5, 4.6, 4.7 This is the difference between the peak temperature of the concrete during hydration (curing) and the ambient surrounding temperature.
Long Term Fall In Temperature ($^\circ\text{C}$) T_2	10	The long term fall in temperature which takes into account the time of year at which the concrete was cast. Cast in UK Summer --> 20°C (typical Value) Cast in UK Winter --> 10°C (typical Value) CIRIA C660 Section 3.2.1, 3.6.1 & 4.3
Coefficient for Creep K_1	0.65	Typical Value is 0.65, CIRIA C660 Section 4.9.1
Restraint Factor During Early Thermal Cycle R_1	0.5	Typical Value is 0.5, CIRIA C660 Section 4.7
Restraint Factor For Long Term Thermal Movement R_2	0.5	Typical Value is 0.5, CIRIA C660 Section 4.7
Restraint Factor For Drying Shrinkage R_3	0.5	Typical Value is 0.5, CIRIA C660 Section 4.7

5.1 - SHORT TERM THERMAL CRACKING		
Autogenous Shrinkage Strain at 3 days $\epsilon_{ca}(3)$	0.000013	BS EN1992-1-1 Eq 3.11 (Also see section 3.2 on this design sheet)
Early Age Restrained Strain ϵ_r	0.074592	$\epsilon_r = K_1 * R_1 * (\alpha_c T_1 + \epsilon_{ca}(3))$ CIRIA C660 - Section 3.2.1
Mean Tensile Strength at 3 days (N/mm ²) $f_{ctm}(3)$	1.655	$f_{ctm}(t) = \beta_{cc}(t) * f_{ctm}$ BS EN1992-1-1 Eq - 3.4
Mean Modulus of Elasticity at 3 days (N/mm ²) $E_{cm}(3)$	24924	$E_{cm}(t) = (f_{cm}(t)/f_{cm})^{0.3} * E_{cm}$ BS EN1992-1-1 Eq - 3.5
Tensile Strain Capacity ϵ_{ctu}	0.075461	$\epsilon_{ctu} = 1.01(f_{ctm}(3) / E_{cm}(3) * 10^{-6}) + 0.0084$ CIRIA C660 - Eq 4.7
Does Thermal Cracking Occur?	NO	Is $\epsilon_r > \epsilon_{ctu}$? (Does restrained strain exceed tensile strain capacity)
Early Age Crack Inducing Strain ϵ_{cr}	N/A	$\epsilon_{cr} = \epsilon_r - 0.5\epsilon_{ctu}$ CIRIA C660 Eq 3.5
5.2 - LONG TERM THERMAL CRACKING		
Autogenous Shrinkage Strain at 28 days $\epsilon_{ca}(28)$	0.0000294	BS EN1992-1-1 Eq 3.11 (Also see section 3.2 on this design sheet)
Drying Shrinkage at Infinite Time $\epsilon_{cd}(\infty)$	0.0004940	Calculated Previously on this design sheet
Long Term Restrained Strain ϵ_r	0.104008	$\epsilon_r = K_1\{[\alpha_c T_1 + \epsilon_{ca}(28)]R_1 + \alpha_c T_2 R_2 + \epsilon_{cd}(\infty)R_3\}$ CIRIA C660 Eq 3.2
Mean Tensile Strength at 28 days (N/mm ²) f_{ctm}	2.766	Calculated Previously on this design spreadsheet
Mean Modulus of Elasticity at 28 days (N/mm ²) E_{cm}	29077	Calculated Previously on this design spreadsheet
Tensile Strain Capacity ϵ_{ctu}	0.104486	$\epsilon_{ctu} = 1.01(f_{ctm}(28) / E_{cm}(28) * 10^{-6}) + 0.0084$ CIRIA C660 - Eq 4.7
Does Thermal Cracking Occur?	NO	Is $\epsilon_r > \epsilon_{ctu}$? (Does restrained strain exceed tensile strain capacity)
Long Term Crack Inducing Strain ϵ_{cr}	N/A	$\epsilon_{cr} = \epsilon_r - 0.5\epsilon_{ctu}$ CIRIA C660 Eq 3.5